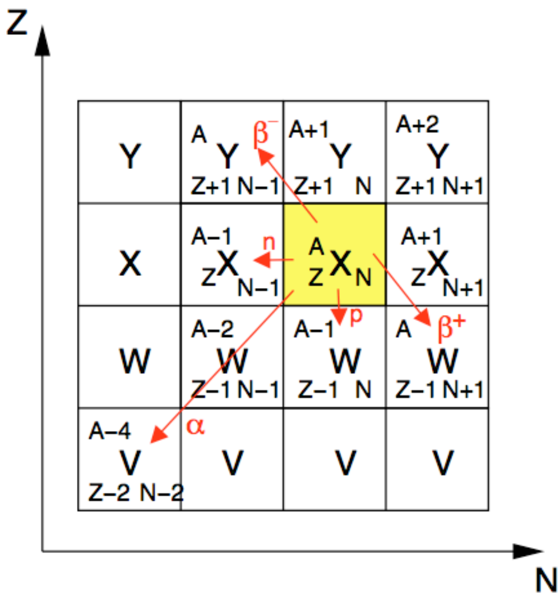
Nuclear decay and radioactivity

Change in nuclear state ⇔ Radiation

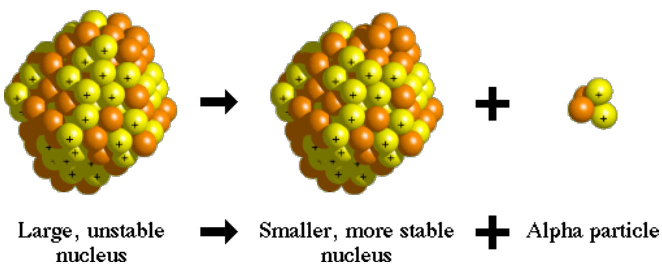
Nuclei decay via:

- α-decay
- β-decay
- γ-decay
  - nuclei have energy levels like atoms, γ-rays are emitted when an nucleus de-excites
  - does not change Z or N
- Nucleon emission
  - Emission of n or p, via a process similar to α-decay
- Spontaneous fission

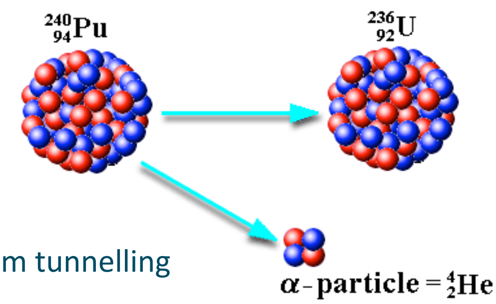


Alpha decay

$^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + {}^{4}_{2}He_{2}$



$^{240}_{94}Pu_{146} \rightarrow {}^{236}_{92}U_{144} + {}^{4}_{2}He_{2}$

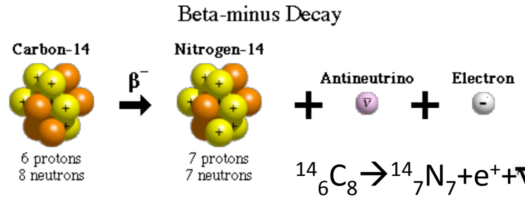


Quantum tunnelling

Beta decays

β^-decay: $n \rightarrow p + e^- + \nu$

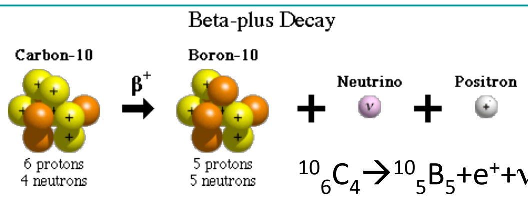
$^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}Y_{N-1} + e^- + \bar{\nu}_e$



$^{14}_{6}C_8 \rightarrow {}^{14}_{7}N_7 + e^+ + \bar{\nu}_e$

β^+-decay: $p \rightarrow n + e^+ + \nu$

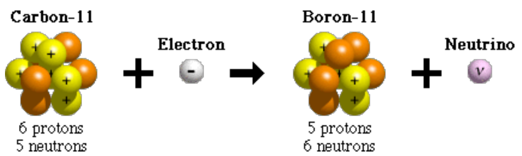
$^{A}_{Z}X_N \rightarrow {}^{A}_{Z-1}Y_{N+1} + e^+ + \nu_e$



$^{10}_{6}C_4 \rightarrow {}^{10}_{5}B_5 + e^+ + \nu_e$

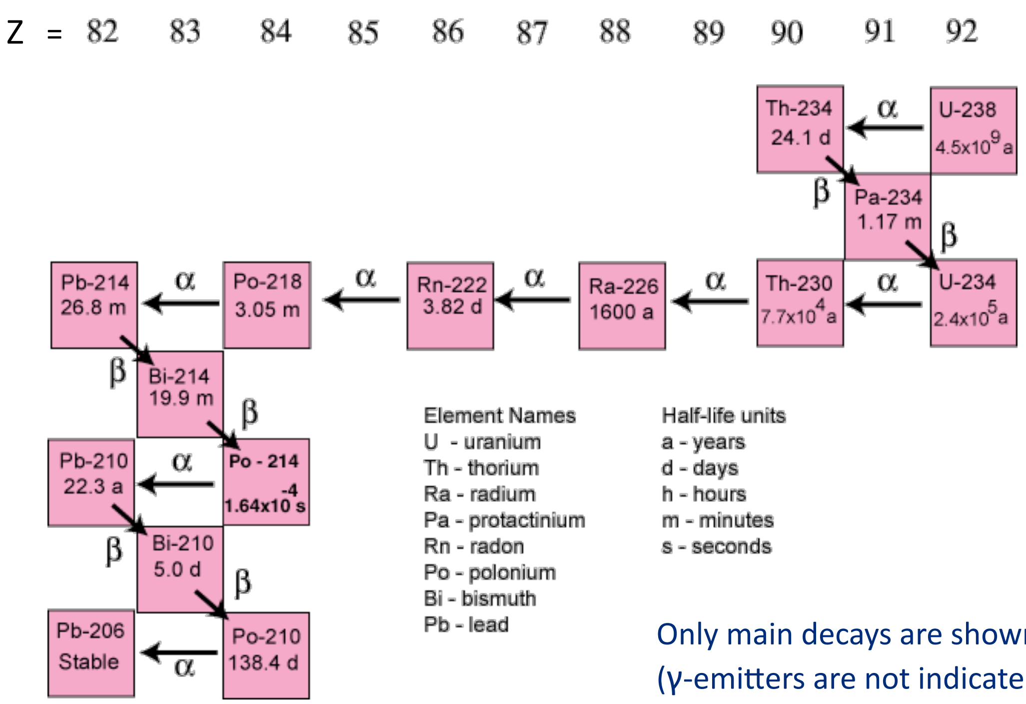
Electron capture: $e^- + p \rightarrow n + \nu$

$e^- + {}^{A}_{Z}X_N \rightarrow {}^{A}_{Z-1}Y_{N+1} + \nu_e$



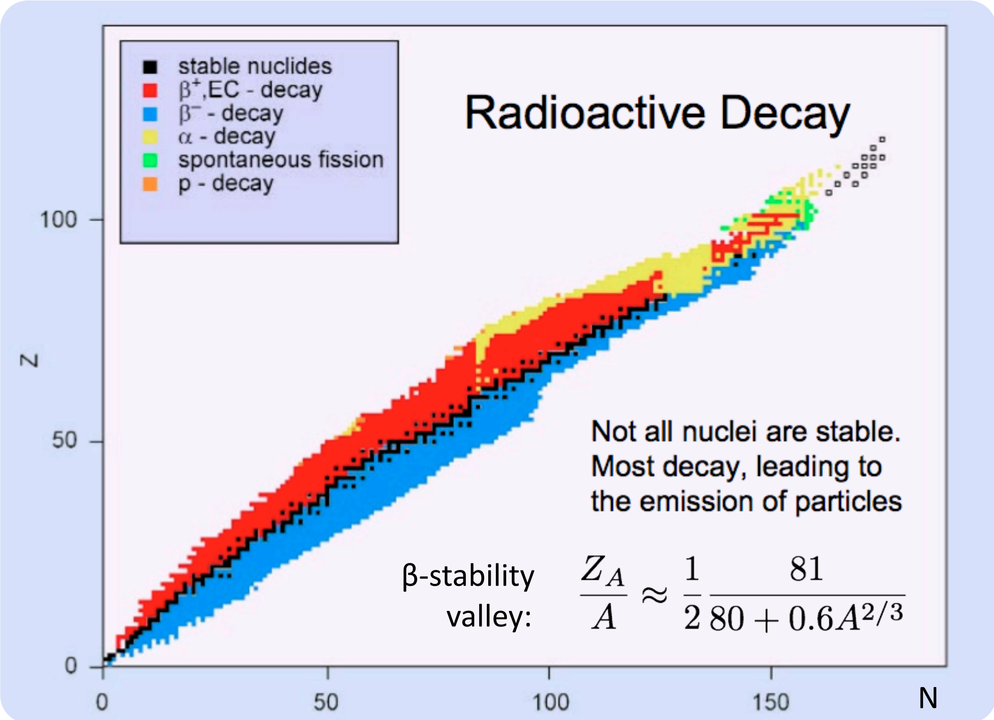
$e^- + {}^{11}_{6}C_5 \rightarrow {}^{11}_{5}B_6 + \nu_e$

The ^{238}U Decay Chain

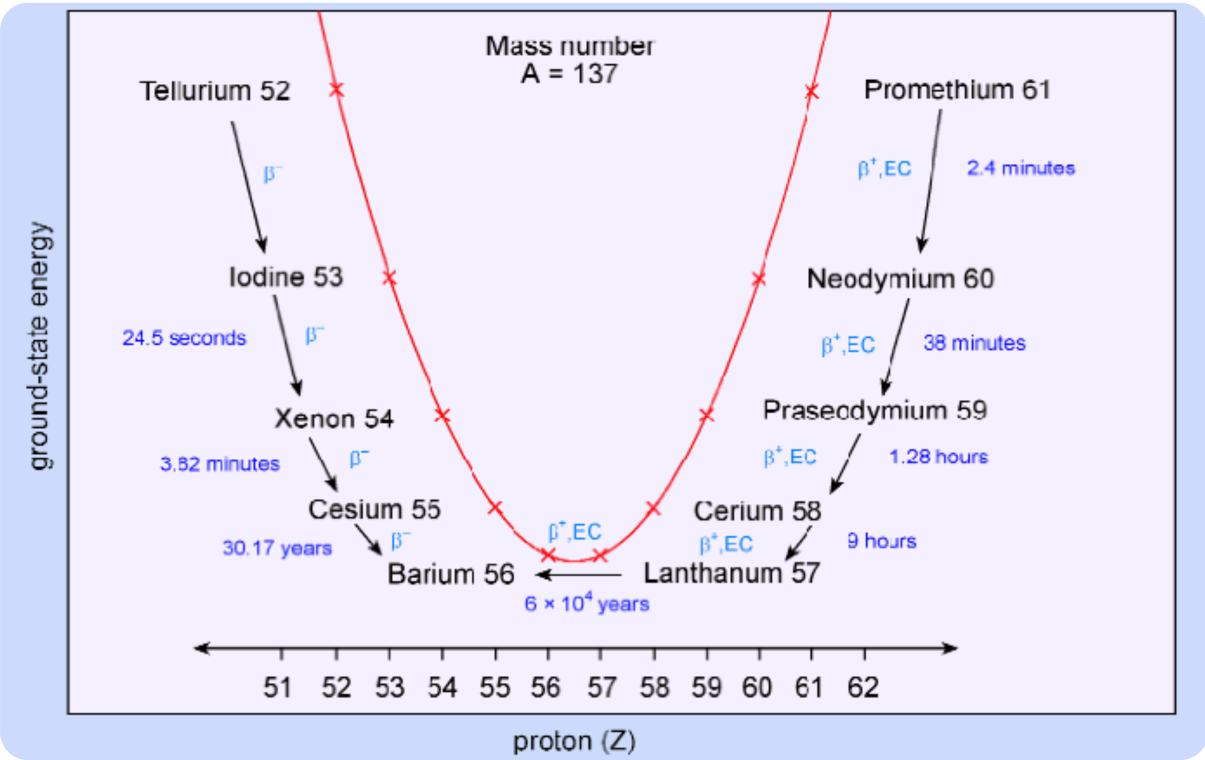


Only main decays are shown
(γ-emitters are not indicated)

What nuclei exist



Stability



Beta-stability valley

Using the Bethe-Weizsäcker semi-empirical formula for the binding energy:

$$B_{tot}(Z,A) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A}$$

$$M(Z,A) = Z \cdot M_p + (A-Z)M_n - B_{tot}(Z,A)$$

$$a_a \frac{(A-2Z)^2}{A} = a_a \frac{A^2 - 4AZ + 4Z^2}{A} = a_a\left(A - 4Z + \frac{4Z^2}{A}\right)$$

$$M = A\left[M_n - a_v + \frac{a_s}{A^{1/3}} + a_a\right] + Z\left[M_p - M_n - 4Za_a\right] + Z^2\left(\frac{a_c}{A^{1/3}} + \frac{4a_a}{A}\right)$$

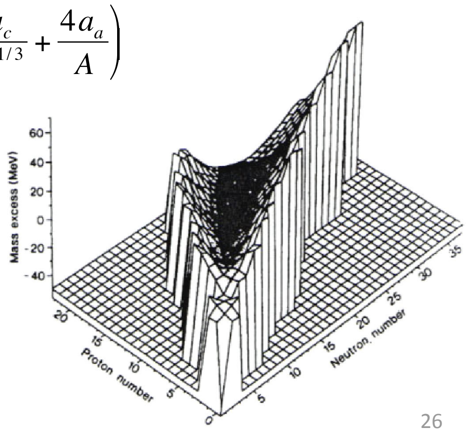
The coefficients a_n are calculated by fitting to experimentally measured masses of nuclei.

This the equation of a parabola *M(Z) = a + bZ + cZ²*

Minimising M:

$$\left(\frac{\partial M}{\partial Z}\right)_A = 0 = b + 2cZ_A$$

$$\frac{Z_A}{A} \approx \frac{1}{2}\frac{81}{80 + 0.6A^{2/3}}$$



Nuclear and Particle Physics
Part 3: Radioactivity

Dr. Dan Protopopescu

Kelvin Building, room 524

Dan.Protopopescu@glasgow.ac.uk

Radioactivity

- Radioactivity is a natural process through which nuclei of unstable elements radiate excess energy in the form of particles
- The underlaying process is called *radioactive decay*
- Radioactive decay is:
  - *spontaneous* - occurs without any interaction* with other atomic constituents
  - a *stochastic* process at the level of single atoms, in that it is impossible to predict when a given nucleus will decay

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* except for decays via *electron capture* or *internal conversion*, when an inner electron of the radioactive atom is involved in the process

The law of radioactive decay

- If a sample of material contains N radioactive nuclei then the number decaying, *dN*, in a time *dt* will be proportional to *N*
- A quantity that decreases at a rate proportional to its value is said to be subject to exponential decay
- N_0 is the number of nuclei at time t=0 and $N(t)$ is the number of nuclei that *have not* decayed by time t

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = -\frac{dN/dt}{N}$$

λ is the decay constant defined as the probability per unit time that a nucleus will decay

$$N(t) = N_0 e^{-\lambda t}$$

How was that derived ?

The number dN of nuclei decaying in a time interval dt will be proportional to N. Mathematically, this is written as (1):

$$\frac{dN}{dt} \propto N$$

If I introduce a constant λ and add a minus sign to take into account the fact that N decreases in time, this can be rewritten as (2):

$$\frac{dN}{dt} = -\lambda N$$

This can be re-arranged as (3):

$$\frac{dN}{N} = -\lambda dt$$

Derivation continued ...

At the time t=0 we have started with N_0 nuclei, so we integrate this with the limits

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{0}^{t} \lambda dt$$

which gives (4)

$$\ln \frac{N}{N_0} = -\lambda t$$

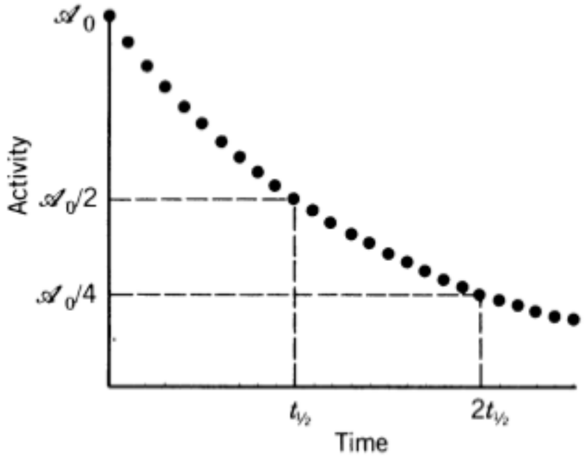
Eq. (4) can be written as

$$N = N_0 e^{-\lambda t}$$

or, if I want to explicitly indicate that N is a <u>function</u> of time

$$N(t) = N_0 e^{-\lambda t}$$

Decay rate and activity



- It is experimentally difficult to directly measure the number of nuclei that have not decayed
- It is more straightforward to measure the activity, *A(t)* of a sample, defined as the number of nuclei decaying per unit time (e.g. as clicks or counts from a Geiger counter in a given time)
- Activity will also follow the exponential decay law with A_0= initial activity = λN_0
- This assumes that we measure over a time *t* that is short compared to $1/\lambda$ ($t \ll 1/\lambda$)

$$A(t) = -\frac{dN(t)}{dt} = \lambda N(t)$$
$$= \lambda N_0 e^{-\lambda t}$$
$$= A_0 e^{-\lambda t}$$

Units of activity

- SI unit of activity is the becquerel (Bq)
  1 Bq = 1 decay/second
- Old unit: the curie (the activity of 1g of radium isotope ^{226}Ra)
  1 Ci = 3.7×10^{10} Bq = 37 GBq
- No account is taken of the type of radiation or how much energy the decay products have

Some examples	
Activity of the radioisotope (e.g. ^{137}Cs or ^{60}Co) in a radiotherapy machine	1000 Ci
Activity of the naturally occurring ^{40}K in the human body	0.1 µCi

Decay probability

- If there are N_0 nuclei at the time *t*, then the number decaying per unit time between *t* and *t+dt* is:

$$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$$

- The probability of a single nucleus decaying in the time interval *dt* is then given by:

$$P_{decay}(t)dt = \frac{1}{N_0}\lambda N_0 e^{-\lambda t}dt = \lambda e^{-\lambda t}dt$$

Mean lifetime τ

In general the *mean* of a variable x that is distributed according to f(x) is given by:

$$\bar{x} = \frac{\int xf(x)dx}{\int f(x)dx}$$

To determine the mean life i.e. the mean time until an unstable nucleus decays we apply:

$$\bar{t} = \tau = \frac{\int_0^\infty t\lambda e^{-\lambda t}dt}{\int_0^\infty \lambda e^{-\lambda t}dt} = \frac{1}{\lambda}$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

$$A(t) = A_0 e^{-\lambda t} = A_0 e^{-t/\tau}$$

The mean lifetime τ of the nucleus is the inverse of the decay constant λ.

Fraction surviving after 1 mean lifetime = e^{-1} = 0.37

after 2 mean lifetimes = e^{-2} = 0.135 etc.

Half-life $t_{1/2}$

- The half-life is the time after which half the sample has decayed:

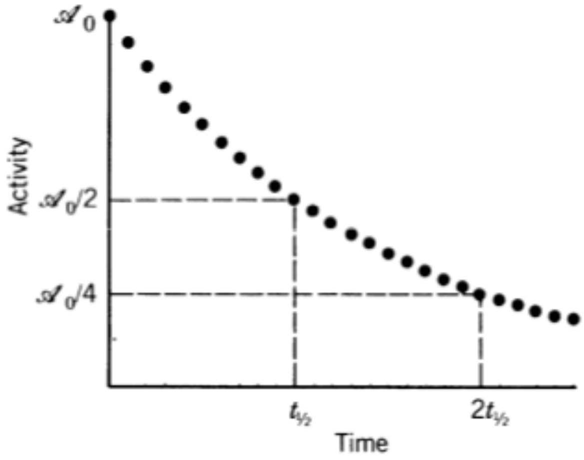
$$\text{When } N = \frac{N_0}{2} \quad t = t_{1/2}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\Rightarrow e^{\lambda t_{1/2}} = 2$$

$$\Rightarrow \lambda t_{1/2} = \ln(2)$$

$$\Rightarrow t_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2$$



Half-life < mean lifetime
Fraction surviving N half-lives=2^{-N}

Recap

Concept	Equation	Definition
Exponential decay	$N(t) = N_0 e^{-\lambda t}$	Number of nuclei that have not decayed by time *t*
Activity	$A(t) = \lambda N_0 e^{-\lambda t}$	Number of nuclei decaying per unit time
Decay probability	$P_{decay}(t) = \lambda e^{-\lambda t}$	Probability of a single nucleus decaying in the interval $t \rightarrow t+dt$
Mean lifetime	$\tau = 1/\lambda$	Mean time until an unstable nucleus decays
Half-life	$t_{1/2} = \ln 2/\lambda$	Time after which half the radioactive sample has decayed