Nuclear and Particle Physics
Part 4: Nuclear Reactions

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Nuclear reactions

- A *nuclear reaction* is defined as the process in which two nuclei, or else a nucleus and a subatomic particle - such as a proton, neutron, or high energy electron, collide to produce products different from the initial particles, e.g.

$$A + B \rightarrow C + D$$

- Radioactive decays can be seen as *spontaneous* nuclear reactions
- Nuclear reactions were first studied by irradiating Po with α-particles and observing induced (or artificial) radioactivity
- Accelerators are now used to accelerate particles, e,p,n,α or even light nuclei, and fire them at different targets. → Many new nuclei have been discovered, shapes of nuclei can be studied etc.

Nuclear reactions

A simple example of a nuclear reaction is firing α-particles at beryllium:

$$\overset{A}{{}^{4}_{2}\alpha} + \overset{B}{{}^{9}_{4}Be} \rightarrow \overset{C}{{}^{12}_{6}C} + \overset{D}{{}^{1}_{0}n}$$

4 + 9 = 12 + 1
2 + 4 = 6 + 0

Projectile **Target** **Products**

This can be equivalently written as:

$${}^{9}_{4}Be(\alpha,n){}^{12}_{6}C$$

In reactions not involving β-decay the total number of neutrons and protons are both conserved:

$$Z_A + Z_B = Z_C + Z_D \qquad N_A + N_B = N_C + N_D$$

Energy released in reactions: Q-value

In a generic reaction:

A	+ B	→ C	+ D	
m_A	m_B	m_C	m_D	(masses)
K_A	K_B	K_C	K_D	(kinetic energies)

Energy conservation condition is written as:

$$m_A c^2 + K_A + m_B c^2 + K_B = m_C c^2 + K_C + m_D c_2 + K_D$$

From this can calculate the *energy released* from the reaction, the so-called *Q*-value:

$$Q = K_{final} - K_{initial} = (K_C + K_D) - (K_A + K_D) \Rightarrow$$

$$\boxed{Q = (m_A + m_B)c^2 - (m_C + m_D)c^2}$$

Example: α-decay

Let us consider the α-decay of ^{226}Ra:

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

and calculate Q with:

$$Q = (m_{Ra226} - m_{Rn222} - m_{\alpha})c^2$$
$$m_{Ra226} = 226.025\,u$$
$$m_{Rn222} = 222.017\,u$$
$$m_{He4} = 4.002\,u$$
$$\Rightarrow \quad Q = (226.025 - 222.017 - 4.002)u \times 931.5\,MeV/u = \underline{5.589\,MeV}$$

We notice that $Q>0$. This energy is converted into the kinetic energies of the daughter nucleus and the α-particle.

Energy released in reactions

If Q>0 mass or binding energy is converted into kinetic energy of the final products, i.e. the reaction is exothermic.

If Q<0 the initial kinetic energy is converted into mass or binding energy, and the reaction is called endothermic.

For reactions where the number of protons and neutrons are both conserved we can use the mass defect formula:

$$Mass\ defect = m_x - A \cdot u$$

where the nucleus X has mass number A.

For endothermic reactions with Q<0, i.e. where mass is created, there is a *threshold (or minimum*) energy that is required to conserve energy and momentum.

Threshold energy

Working in the lab frame where A is the beam particle and B is a stationary target:

	A	**+**	**B**	**→**	**C**	**+**	**D**
Mass	m_A		m_B		m_C		m_D
KE	K_A		0		K_C		K_D
Momentum	p_A		0		p_C		p_D

To find threshold energy we transform to the centre of mass frame (where total momentum $p_{tot}=0$). Assuming non-relativistic velocities, i.e. $v_A \ll c$

	Lab		CM	
	A	B	A	B
Velocity	v_A	0	$v_A - u$	$-u$
Momentum	$m_A v_A$	0	$m_A(v_A - u)$	$-m_B u$

Threshold energy

The velocity of the cm frame is: $u = \frac{m_A}{m_A + m_B} v_A$

The threshold energy is defined in the CM system for A+B→C+D by the condition that C and D are produced with zero kinetic energy:

$$m_A c^2 + \frac{1}{2} m_A (v_A - u)^2 + m_B c^2 + \frac{1}{2} m_B u^2 = m_C c^2 + m_D c^2$$

$$\Rightarrow \quad \frac{1}{2} m_A (v_A - u)^2 + \frac{1}{2} m_B u^2 = -Q$$

The threshold kinetic energy K_{th} in the lab system is given by:

$$K_{th} = \frac{m_A v_A^2}{2}$$

By eliminating u we obtain: $K_{th} = -\left(1 + \frac{m_A}{m_B}\right) Q$

Examples

1) $p + {}^{7}_{3}Li \rightarrow {}^{4}_{2}He + {}^{4}_{2}He$

$$Q = (m_p + m_{Li} - 2m_\alpha)c^2$$

$$m_p = 1.007825\,u, \quad m_{Li} = 7.01600\,u,$$

$$m_\alpha = 4.0026\,u, \quad 1uc^2 = 931.502\,MeV$$

$$\Rightarrow \quad Q = (1.0078 + 7.01600 - 2 \times 4.0026)u \times 931.5\,MeV/u = 17.32\,MeV$$

2) ${}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$ or ${}^{14}_{7}N(\alpha, p){}^{17}_{8}O$

$$Q = (m_\alpha + m_N - m_O - m_p)c^2$$

$$m_N = 14.003074u, \quad m_O = 16.999131u$$

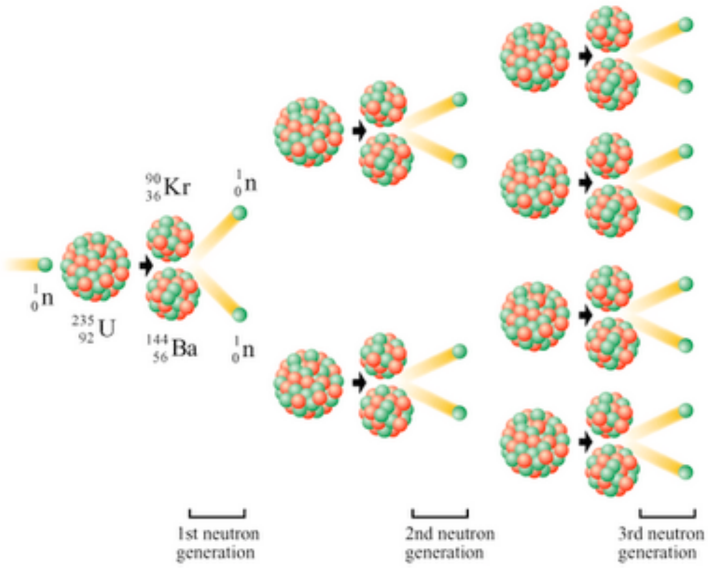
$$\Rightarrow \quad Q = (4.0026 + 14.0031 - 16.9991 - 1.0078)u \times 931.5\,MeV/u$$

$$= -1.178\,MeV < 0$$

$$K_{th} = -(1 + m_\alpha / m_N)Q = (1 + 4/14)Q = 1.514\,MeV$$

Nuclear Fission

- Nuclear fission releases energy by splitting heavy elements
- At a power plant, the energy produced from a controlled fission chain-reaction generates heat to produce steam, which is then used in a turbine to generate electricity
- In a nuclear bomb, the reaction happens very quickly and the energy released manifests itself as an explosion



Energy released

Let us take the induced fission reaction (with thermal neutron):

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2\,{}^{1}_{0}n$$

and calculate Q with:

$$Q = (m_{U235} + \cancel{m}_n - m_{Ba144} - m_{Kr90} - \cancel{2}m_n)c^2$$

$$m_{U235} = 235.044\,u = 218941\,MeV/c^2$$

$$m_{Ba144} = 143.92\,u = 134060\,MeV/c^2$$

$$m_{Kr90} = 89.92\,u = 83759\,MeV/c^2$$

$$m_n = 939\,MeV/c^2$$

$$\Rightarrow \quad Q = 218941 - 134060 - 83759 - 939 = \underline{183\,MeV}$$

Tiny amount ?

- One mole of substance contains $N_A = 6.023\times10^{23}$ atoms.
- In one kilogram of ^{235}U there are

  $\nu = 1000/235 = 4.255$ moles
- That means 1kg of ^{235}U contains

  $N = \nu N_A = 4.255 \times 6.023\times10^{23} = 2.56\times10^{24}$ nuclei
- If only a fraction $f = 0.8$ of them participate in the chain reaction and fission the energy released is

  $Q_{tot} = fNQ = 0.8 \times 2.56\times10^{24} \times 183 = 3.75\times10^{26}\,MeV$
- Given that $1MeV = 1.602\times10^{-13}\,J$, this translates to

  $Q_{tot} = 1.602\times10^{-13} \times 3.75\times10^{26} = 6.00\times10^{13}\,J = \underline{60\,TJ}$
- How much mass has actually been transformed into energy ?

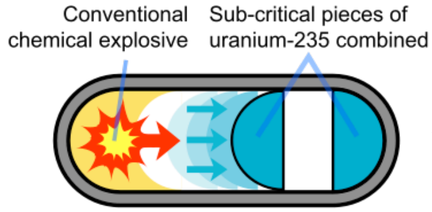
  $m = 6\times10^{13} / (3\times10^{8})^2 = 0.67\times10^{-3}\,kg = 0.67\,g$!!

Energy release comparisons

Event	Type	TNT Equivalent	Energy released
Tsar Bomba	3-stage Teller-Ulam design Thermonuclear test bomb	50 MT	210,000 TJ
Castle Romeo	Thermonuclear test bomb	15 MT	63,000 TJ
Chernobyl (one unit) during one year	RBMK-1000 reactor 1GWe (3.2 GWt)		30,000 TJ
Three Mile Island Unit 1 operated one year	PWR reactor 800 MWe		24,000 TJ
Fukushima Unit 1 operated one year	BWR3 reactor (GE) 439 MWe		14,000 TJ
Peacekeeper	ICBM weapon	10 x 300 kT	12,500 TJ
Fat Man (Nagasaki)	^{239}Pu implosion-type bomb	21 kT	88 TJ
Little Boy (Hiroshima)	^{235}U gun-type bomb	15 kT	64 TJ

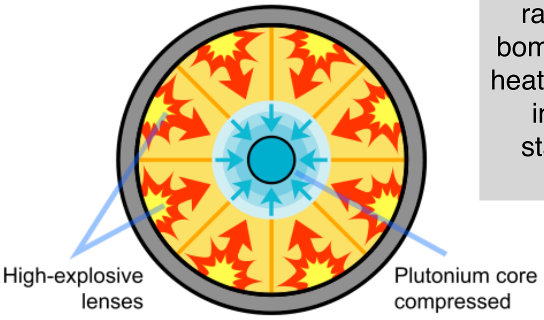
Nuclear weapons

Fission bomb



Gun-type assembly method

Little Boy (Hiroshima)

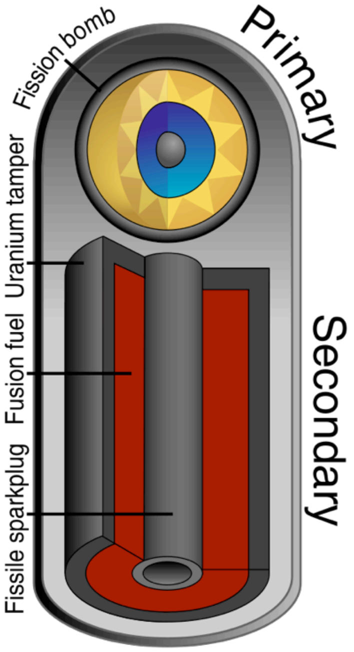


Implosion assembly method

Fat Man (Nagasaki)

Thermonuclear bomb

Teller–Ulam design: the radiation from fission bomb compresses and heats up the fusion fuel initiating the second stage fusion reaction (Tsar Bomba)

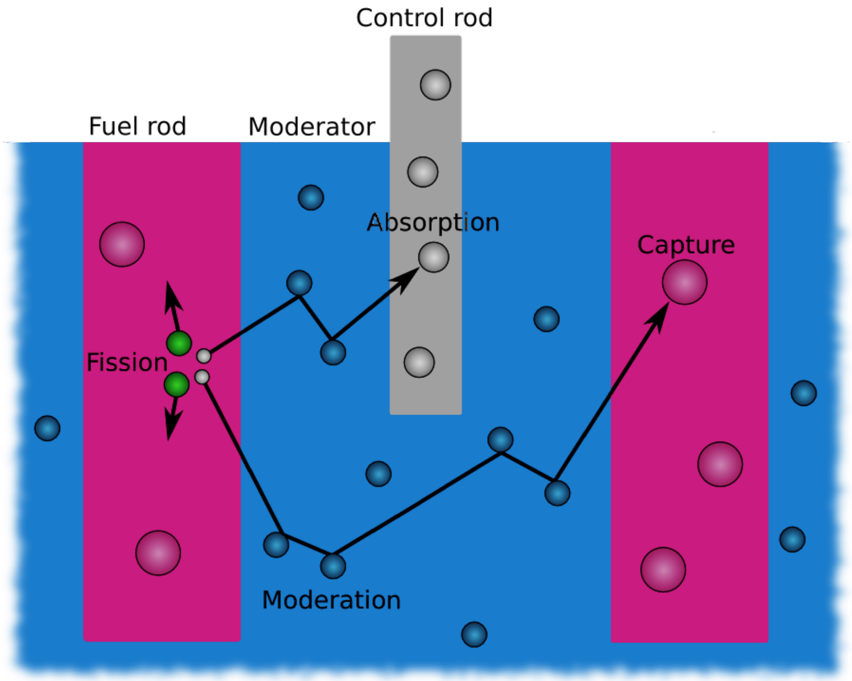


Images source: Wikipedia

Controlled fission chain-reaction

- A ^{235}U nucleus captures a neutron and fissions
- 2-3 fast neutrons are released
- the neutrons are slowed down in the moderator
- any of these neutrons can be absorbed by another ^{235}U nucleus which will fission ...

... and so forth



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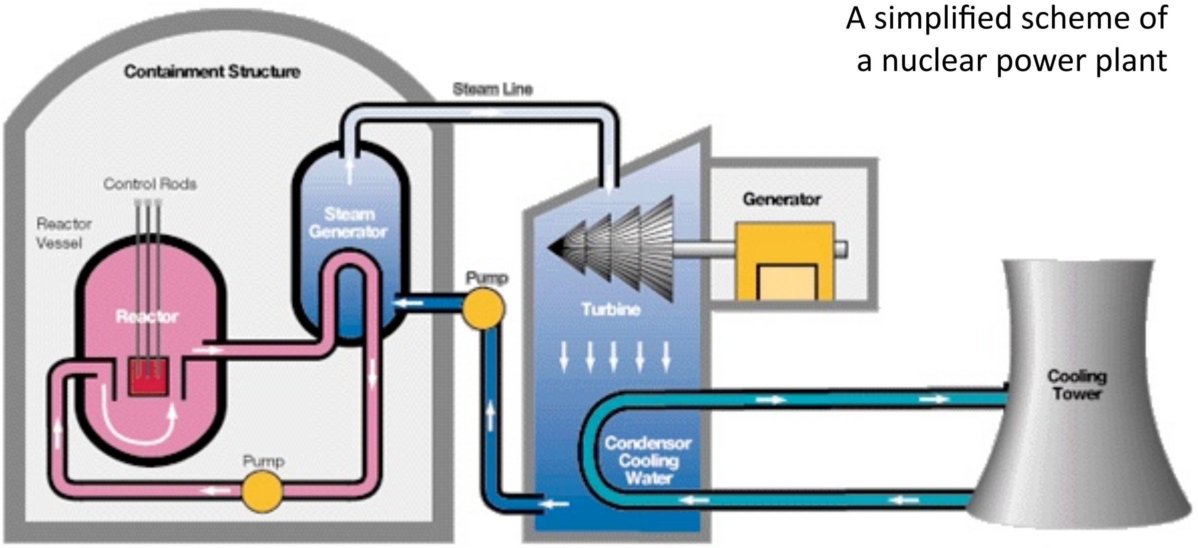
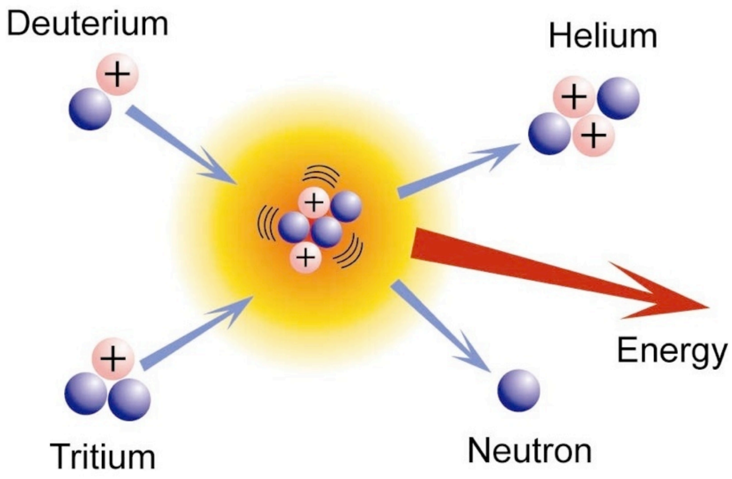
Nuclear power plant

A simplified scheme of
a nuclear power plant



Nuclear fusion

- Nuclear fusion is the process by which two or more atomic nuclei fuse together to form a single heavier nucleus
- Accompanied by release or absorption of large quantities of energy
- The fusion of nuclei with mass lower than iron (Fe) releases energy
- Fusion is the process that powers active stars or the hydrogen bomb



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Fusion reactors

- Imply none of the complexities associated with handling radioactive materials
- Does not pose a radiation threat, as the plasma will immediately vanish if the reaction process goes awry
- Based on the reaction

$$ {}^{2}_{1}D + {}^{3}_{1}T \rightarrow {}^{4}_{2}He + {}^{1}_{0}n $$

- Deuterium is relatively abundant and tritium can be mass-produced in nuclear reactors by enclosing the plasma in a breeder blanket of lithium
- Since the 1950s, several fusion reactors have been built, but as yet none has produced more thermal energy than electrical energy consumed
- There are many technical challenges

Energy released

Let us take the fusion reaction:

$$ {}_{1}^{2}D + {}_{1}^{3}T \rightarrow {}_{2}^{4}He + {}_{0}^{1}n $$

and calculate Q with:

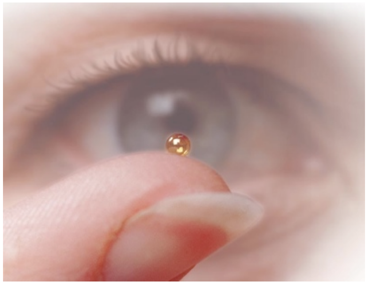
$$ Q = (m_D + m_T - m_{He} - m_n)c^2 $$

$$ \left.\begin{array}{l} m_D = 2.0140u = 1876.02\ MeV/c^2 \\ m_T = 3.0160u = 2809.37\ MeV/c^2 \\ m_{He} = 4.0026u = 3728.38\ MeV/c^2 \\ m_n = 939.56\ MeV/c^2 \end{array}\right\} \Rightarrow $$

$$ Q = 1876.02 + 2809.37 - 3728.38 - 939.56 = \underline{17.44\ MeV} $$

Inertial confinement fusion

- Where the nuclear fusion reactions are initiated by heating and compressing a fuel target containing a mixture of deuterium and tritium
- Fuel micro-balloons filled with either D and T gas or DT ice
  - Energy is delivered to the outer layer of the target using high-energy lasers (1)
  - The heated surface explodes and compresses the inside fuel (2)
  - During the final part of the capsule implosion, the fuel core reaches 20 times the density of Pb and ignites at 100,000,000˚C (3)
  - Thermonuclear burn spreads rapidly through the compressed fuel, yielding many times the input energy (4)



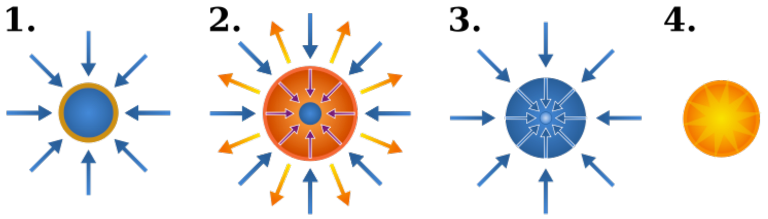
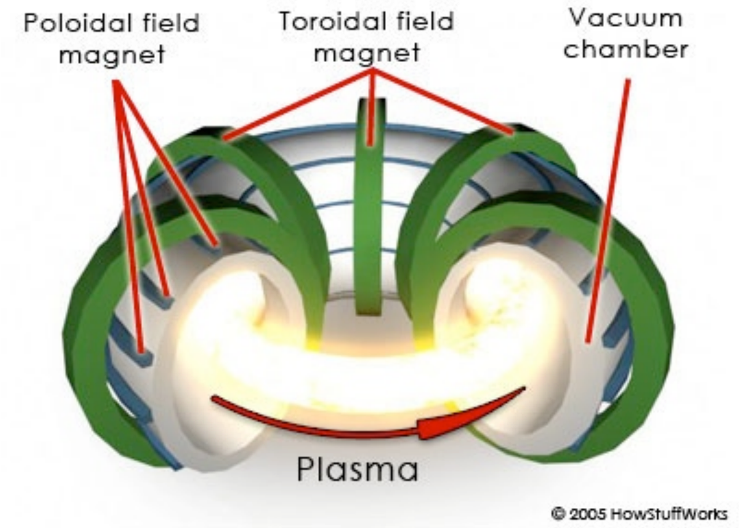


Image source: Wikipedia

The tokamak technology

- Research pioneered by Russians in the late 1950s; First successful test in 1968
- Uses a magnetic field to confine a D+T plasma to a toroidal shape and keep it away from the containment wall
- Stable plasma equilibrium is achieved via a combination of toroidal and poloidal fields
- Plasma high temperatures are achieved via ohmic heating, neutral beam injection or via RF or microwave heating
- Solid vacuum vessel acts also as shielding
- Energy is extracted from the escaping neutrons and transferred to a primary coolant

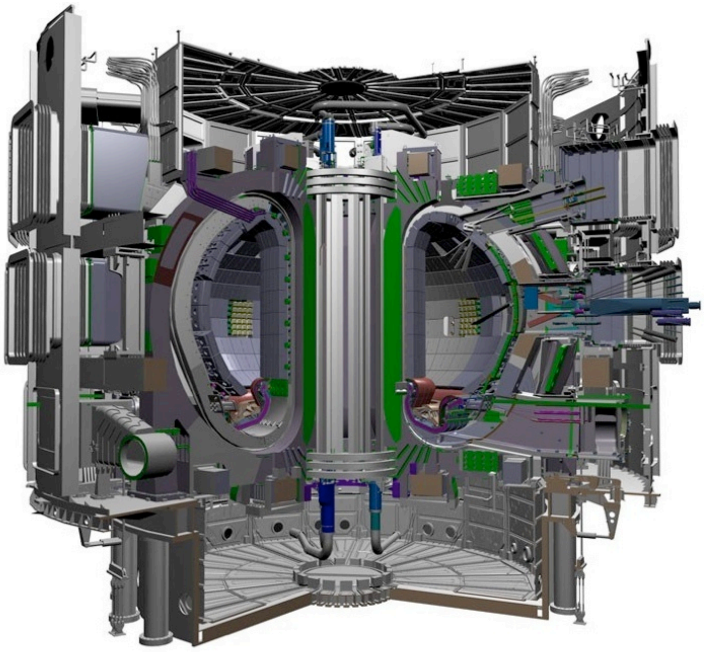


Fusion for power generation

- Advantages:
  - virtually zero pollution
  - produces hundreds of times less radioactive waste than that of a fission reactor
  - no long-lived radioactive waste
  - a large-scale runaway *chain reaction* is impossible. Direct contact with the walls of the reactor chamber would contaminate the plasma, cooling it down immediately and stopping the fusion process
  - the amount of fuel planned to be contained in a fusion reactor chamber is very small (at any given time, ITER’s plasma would contain ~0.5g of D+T fuel, only enough to sustain the reaction for about one hour)
  - an accident would release very small amounts of radioactive material
- The technical challenges are to:
  - maintain a stable plasma configuration
  - find materials that can withstand the *intense neutron fluxe*s produced
  - extract energy for useful purposes
  - produce sizeably more energy than what is put in

ITER

- Currently leading the effort to commercialise fusion power
- ITER Organisation was established in October 2007 but project dates from 1985
- Members: China, EU, India, Japan, S. Korea, Russia, USA
- The machine is being built now at Cadarache, France
- Project timescale: 30 years
- Maximum power: 500MW
- Fusion energy gain factor will be between 5 and 10
- *iter* is the Latin for 'journey'



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