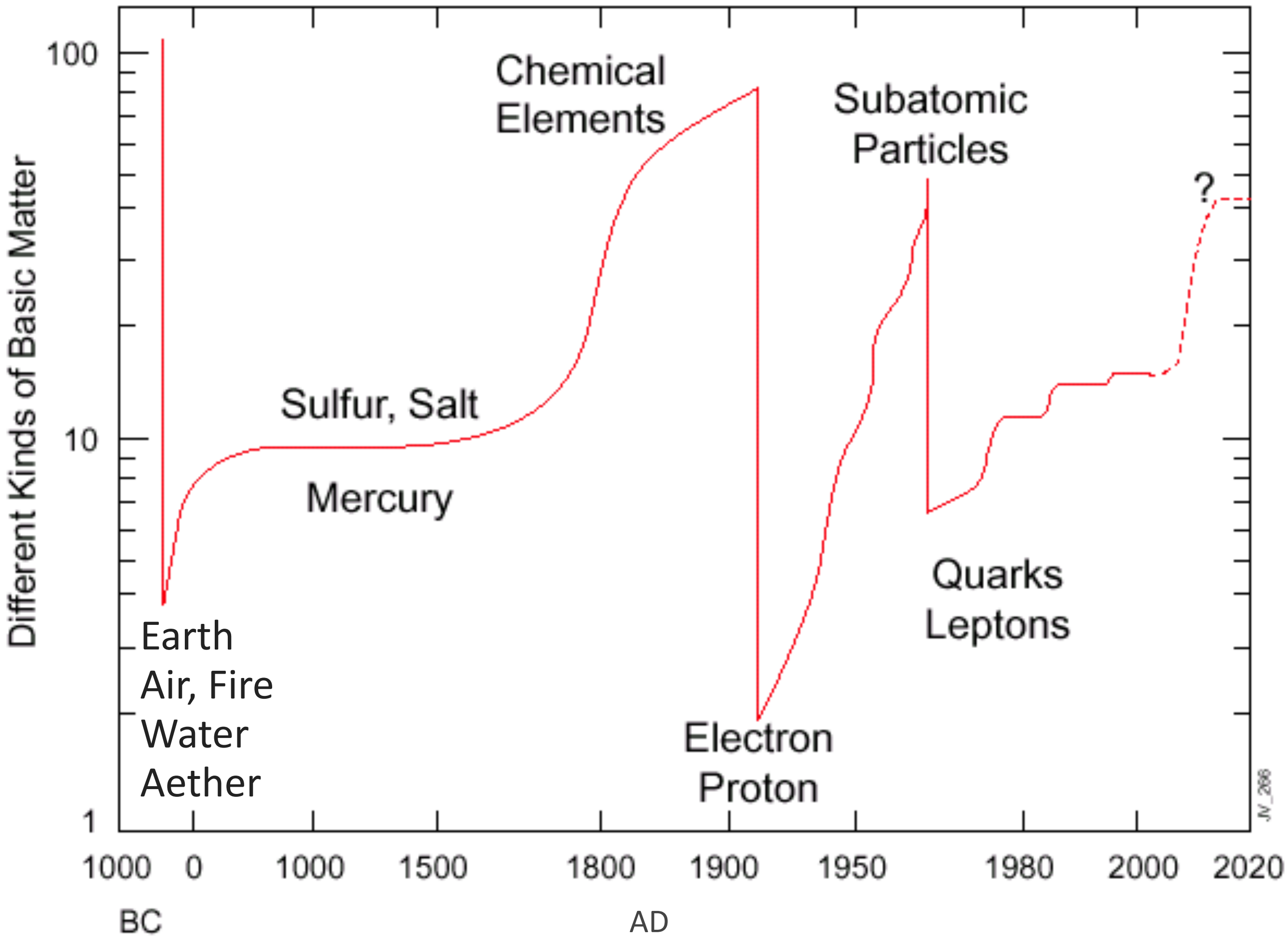
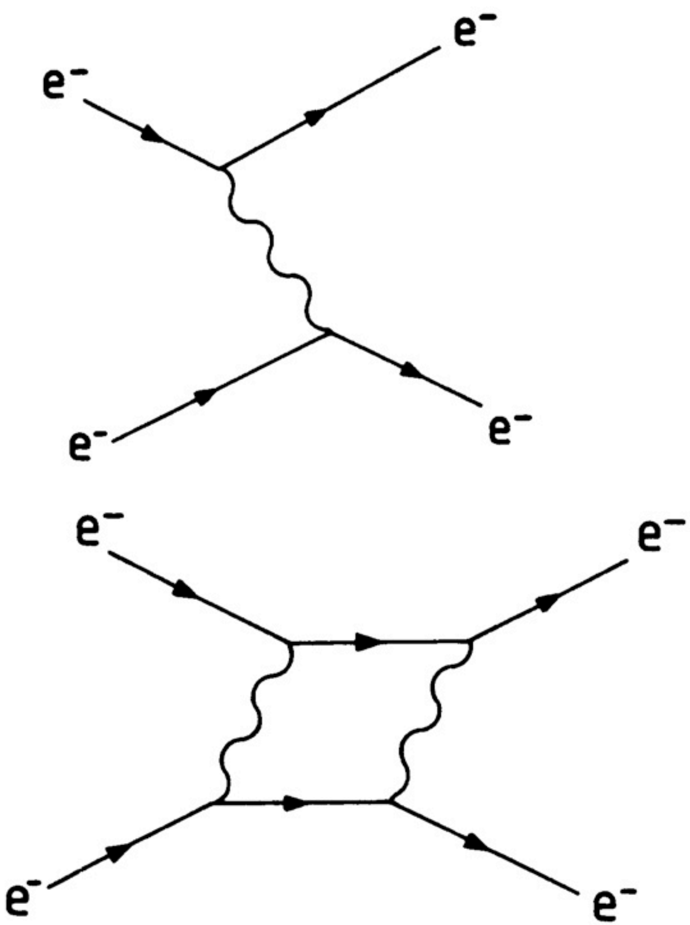
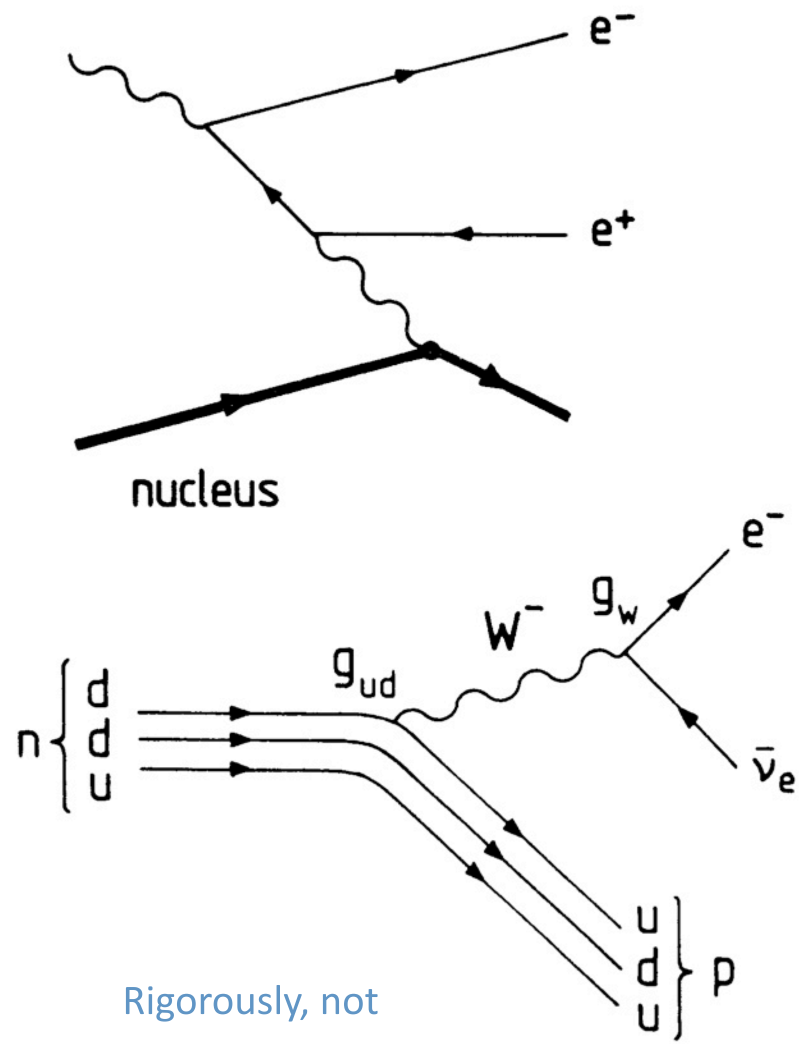
Constituents of matter



True Feynman diagrams



Feynman diagrams



Rigorously, not

Nuclear and Particle Physics

Part 6: Special Relativity

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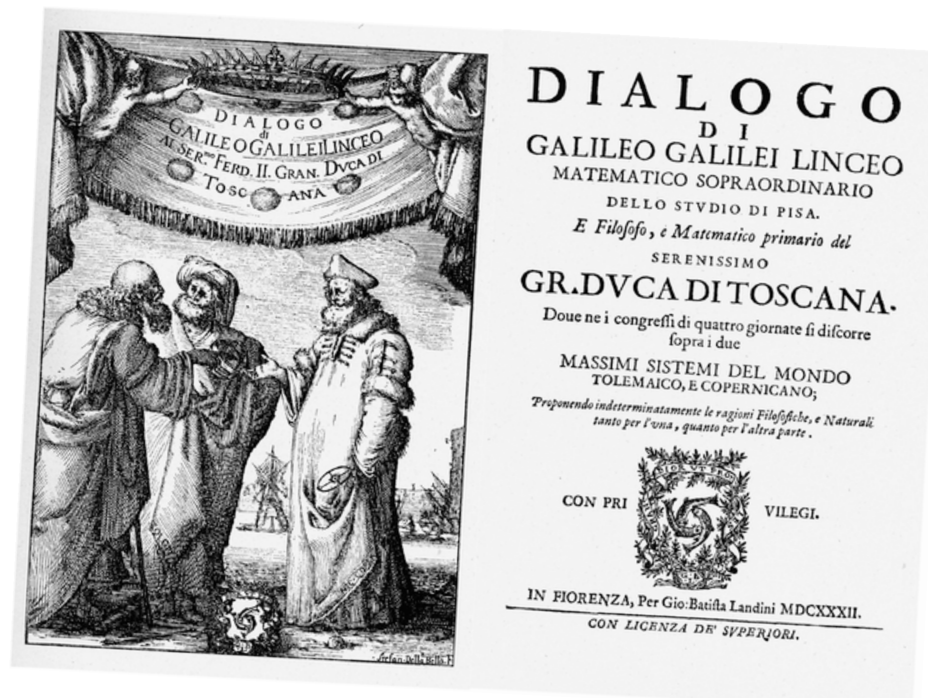
Classical relativity

The principle of relativity was first stated by Galileo Galilei in 1632 in his "*Dialogue concerning the two chief world systems*".

The book contains a thought experiment in which a sailor below decks on a ship cannot tell whether the ship is docked or is moving smoothly through the water: he observes water dripping from a bottle, fish swimming in a tank and so on, and all happen just the same whether the ship is moving or not.

This is a classic exposition of the inertial frame of reference concept and refutes the objection raised at that time that if one was moving as the Earth rotated, anything that one dropped would have to fall behind and drift to the west.



Galilean transformations

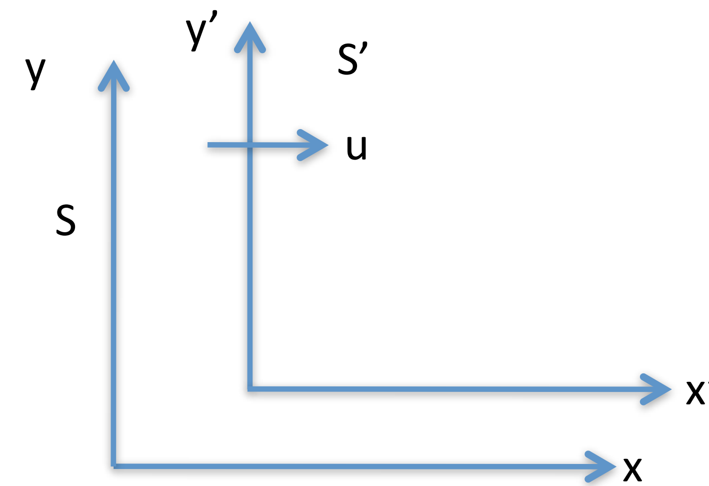
Two inertial coordinate systems (the systems S and S' in the figure) in constant relative motion (with velocity u) along the x-direction are related to one another according to the Galilean transformation

$$x' = x - ut$$

$$t' = t$$

This is the simplest case. In general one should include y, z, and translations and rotations in 3D.

Velocities transform with: $v'_x = v_x - u$



This governs our everyday experience

Inertial frame of reference - definition

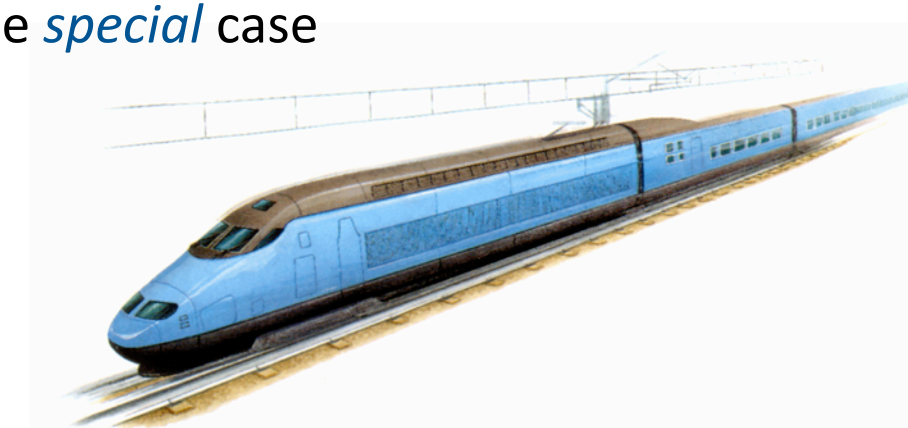
- An *inertial frame* is one in which Newton's laws hold:
  - The frame is either at rest or moving at a constant velocity

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 0$$

- An accelerating frame is a *non-inertial* frame
  - In a non-inertial frames there are forces due to the acceleration of the reference frame

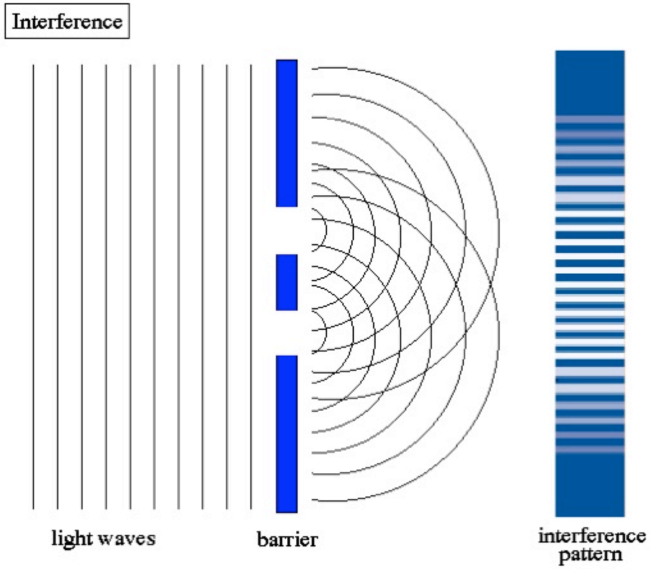
$$\vec{a} = \frac{d^2\vec{r}}{dt^2} \neq 0$$

- Special relativity deals with the *special* case of inertial reference frames



The ether

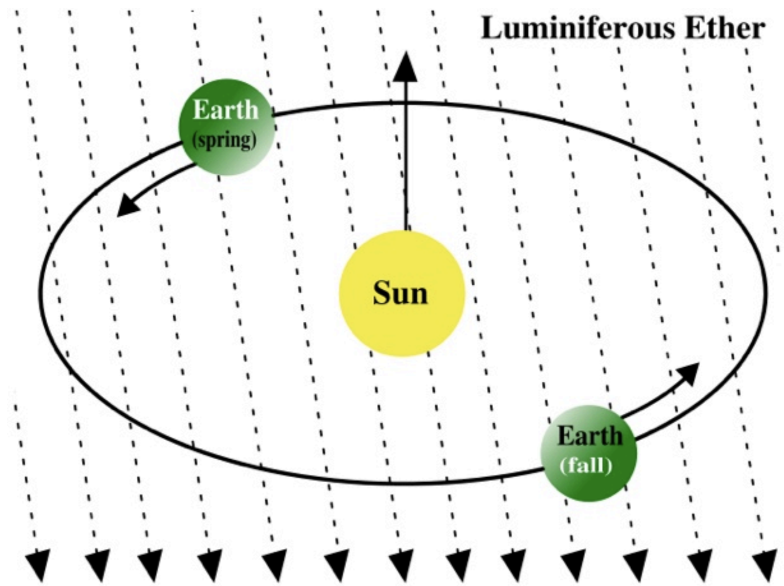
- However, new experiments and discoveries were complicating the facts
- Mathematically, light was found to behave like a wave
- Waves transmit energy through displacements of a medium:
  - Sound waves: gas
  - Earthquakes: rock
  - Ripples: water etc.
- Logically, there must’ve been a medium through which light propagates



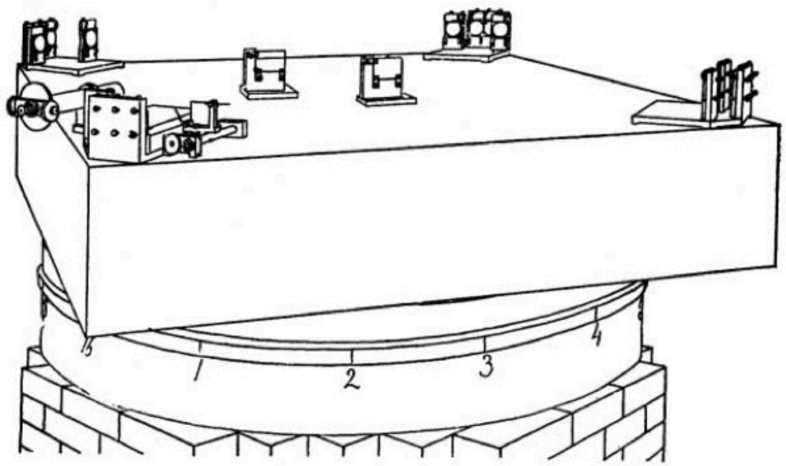
The Properties of Ether

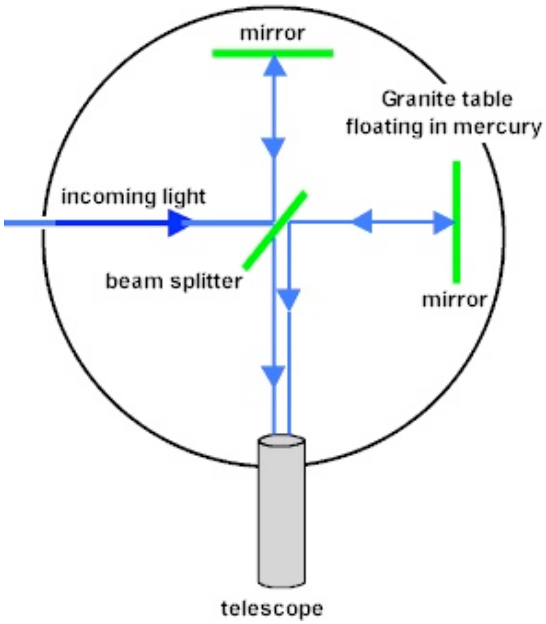
In the late 19th century, luminiferous ether, meaning light-bearing aether, was the term used to describe a medium for the propagation of light. The word ether (aether or æther) stems via Latin from the Greek αιθήρ, which was one of Aristotle's 5 elements.

Electromagnetic waves had been postulated by Maxwell and experimentally detected by Hertz. Light, as an electromagnetic wave, was expected to travel in a medium like sound waves travel through air. As the solar system moved through the ether and the Earth moved around the Sun, an effect on the speed of light from this movement was generally expected.

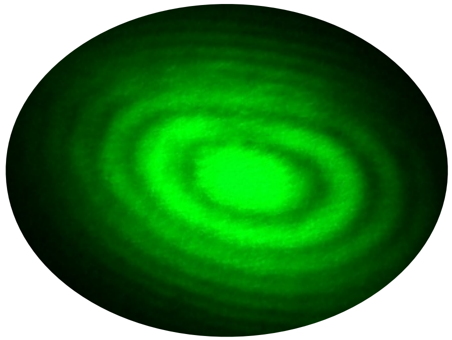


The Michelson-Morley Experiment





The telescope in a Michelson-interferometer sees a pattern of interference fringes. If there would be an effect from the Earth movement through the ether, the position of these fringes would shift. The original experiment was sensitive to a fringe-shift of about 0.01 fringes, while the expected effect was 0.4 fringes. No effect was observed.



The original article from the American Journal of Science (vol. 35, 1887, p. 333-45) can be found at

http://www.aip.org/history/gap/Michelson/02_Michelson.html

Lorentz Contraction

To explain the inconsistency between Galilean relativity and Maxwell's electrodynamics shown by the Michelson-Morley experiment, Hendrik Lorentz suggested that a rigid body should contract by a factor of

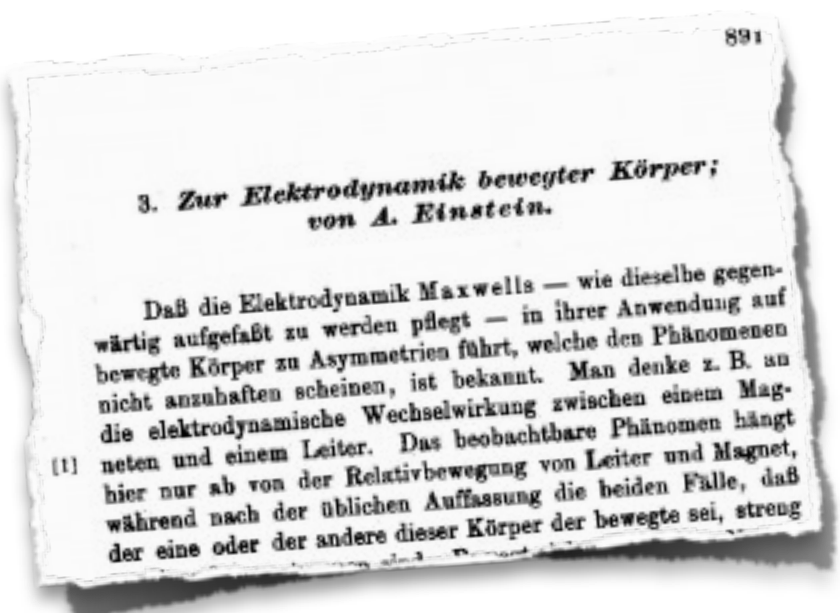
$$\sqrt{1-\frac{v^2}{c^2}}$$

along the direction of its motion. This was supposed to be an effect of the motion through the ether on the electromagnetic forces between the particles making up the body. This length contraction would exactly cancel the effect of the change in the speed of light.

H. Lorentz received a Nobel Prize in 1902 for the theoretical explanation of the Zeeman effect. The Lorenz force is also named after him.

Einstein's contribution

Albert Einstein, at that time as an examiner at the Patent Office in Bern, Switzerland, wrote in 1905 (his '*Annus Mirabilis*') a paper in which he proposed a theory that could solve this impasse: "*On the electrodynamics of moving bodies*" appeared (in German) in Annalen der Physik 17 (10): 891–921.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3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Mag-
[1] neten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng


A. Einstein in 1905

Special relativity: Postulates

In this paper Einstein stated the two postulates of Special Relativity

- **First postulate:**
  The laws of physics are the same in all inertial reference frames

- Throwing a ball on a train looks the same whether the train is at rest or moving at a constant velocity (but not accelerating!)
- An induced electromagnetic fields can be due to the magnet moving or the coil moving
- As Maxwell's laws of electrodynamics are the same in all reference frames it means that the speed of light is the same in all reference frames

- **Second postulate:**
  The speed of light is the same in all inertial frames regardless of the velocity of the source or the observer.

Simplified derivation

What if we add additional terms to the Galilean transformations ?

$$\left.\begin{aligned} x' &= x - ut \\ t' &= t \end{aligned}\right\} \rightarrow \begin{cases} x' = Ax + Bt \\ t' = Cx + Dt \end{cases}$$

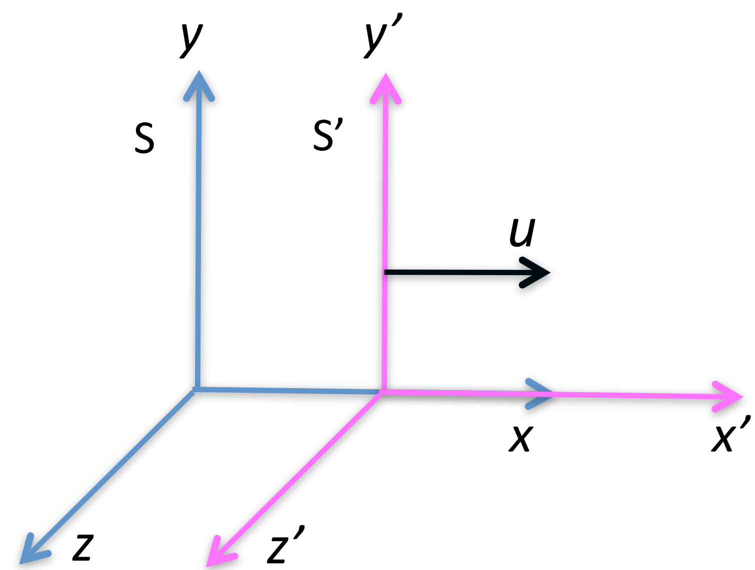
For a point that is at rest in S (x=constant)

$$\frac{dx'}{dt'} = -u = \frac{B}{D} \quad \Rightarrow \quad B = -uD$$

For a point at rest in S' (x'=constant)

$$\frac{dx}{dt} = u = -\frac{B}{A} \quad \Rightarrow \quad B = -uA$$

and then $D=A$



Simplified derivation (cont.)

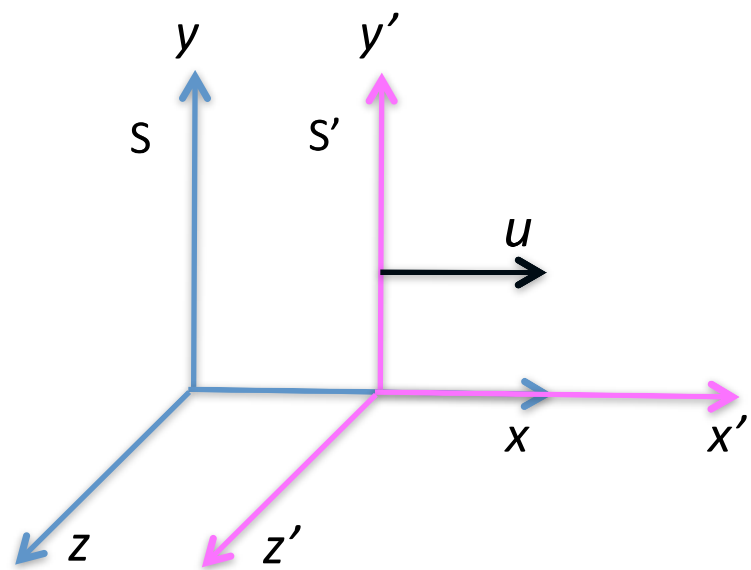
So we've got so far (while leaving out $y'=y$ and $z'=z$) the equations:

$$\begin{cases} x' = A(x - ut) \\ t' = Cx + At \end{cases} \qquad (1)$$

From the second postulate, it follows that light should propagate like a spherical wave in both frames. If we assume that wave started at $t=0$ when the the origins of the two frames were overlapping

$$x^2 + y^2 + z^2 = c^2 t^2 \qquad (2)$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \qquad (3)$$



Now if we introduce x' and t' from (1) into equation (2) we obtain

$$A^2 (x - ut)^2 + y^2 + z^2 = c^2 (Cx + At)^2$$

Simplified derivation (cont.)

If we expand this equation, make use of (2) and isolate the terms in x^2, t^2, and xt

$$A^2 - 1 = c^2 C^2$$

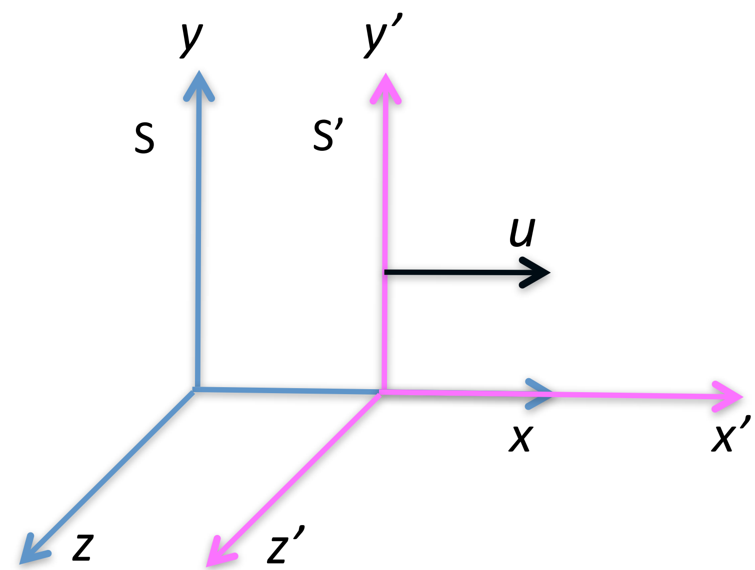
$$A^2 u^2 = (A^2 - 1)c^2$$

$$-2A^2 u = 2c^2 AC$$

From these equations we can extract the coefficients

$$A = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$C = -\frac{u}{c^2} A$$



Lorentz transformations

We have obtained the ransformations between two sets of coordinates of events (x,y,z,t) and (x',y',z',t') measured in frames S and S' respectively where:

- S' moves at velocity u with respect to S along the common x-x' axis
- The origins of both reference frames coincide at $t=t'=0$
- (x,y,z,t) and (x',y',z',t') are space-time coordinates

$$x' = \frac{x - ut}{\sqrt{1-\beta^2}} \quad \text{where} \quad \beta = \frac{u}{c}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \beta x / u}{\sqrt{1-\beta^2}}$$

Velocity transformation:

$$v_x' = \frac{dx'}{dt'} = \frac{v_x - u}{1 - \beta v_x / u} \qquad (5)$$

Classical limit

At low velocities, we have to recover the Galilean transformations:

$$u \ll c \quad \Rightarrow \quad \beta \approx 0 \quad \Rightarrow \quad \begin{cases} x' = \lim\limits_{\beta \to 0} \dfrac{x - ut}{\sqrt{1-\beta^2}} = x - ut \\ t' = \lim\limits_{\beta \to 0} \dfrac{t - \beta x / u}{\sqrt{1-\beta^2}} = t \\ v_x' = \lim\limits_{\beta \to 0} \dfrac{v_x - u}{1 - \beta v_x / u} = v_x - u \end{cases}$$

(compare those with the transformations on slide 5).

Relativistic momentum

- According to Einstein's first postulate the laws of physics must be the same in all inertial reference frames.
- The conservation of linear momentum states that *the total momentum of two colliding objects is constant* provided there are no external forces.
- A collision in frame S is measured and momentum is conserved as expected.
- What happens in the other inertial frame S'?

- In S, the momentum of a particle moving at velocity v is given by:

$$p = mv$$

where m is the mass of the particle

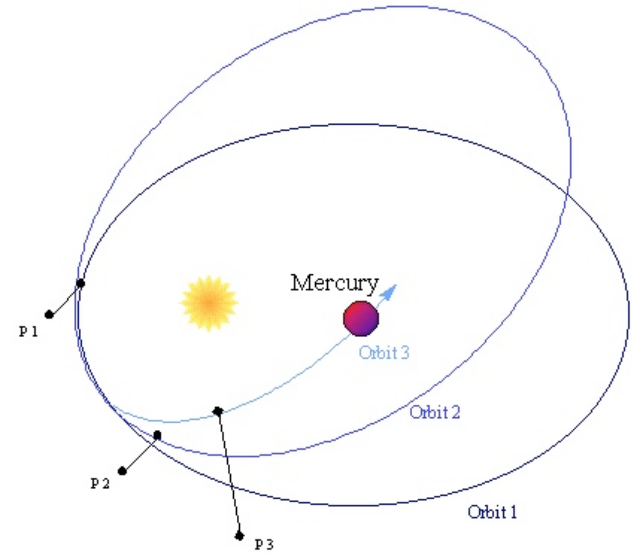
- In the frame S' we must transform the velocities according to the Lorentz velocity transformation (5).
- By formula $p = mv'$ the total momentum is no longer conserved.
- To maintain conservation of momentum, one must define the relativistic momentum as:

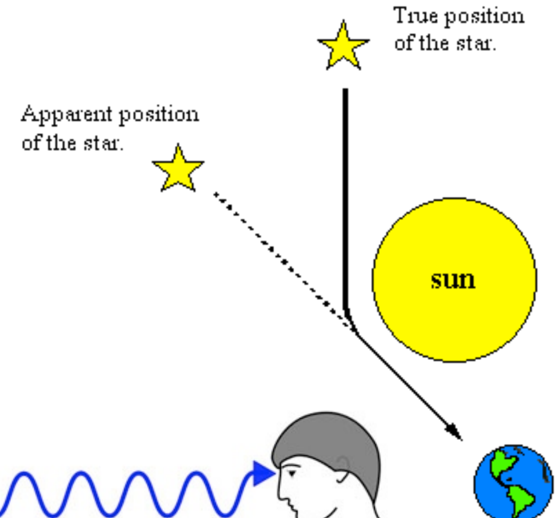
$$p = \frac{mv}{\sqrt{1-\beta^2}}$$

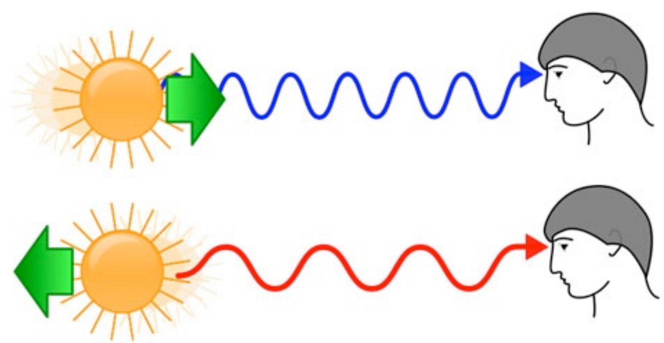
More on this later

Experimental evidence

- Relativity (not limited to the 'Special' part) has been confirmed by many experiments
- Early confirmations:
  - the perihelion precession of Mercury's orbit
  - the deflection of light by the Sun
  - the gravitational redshift of light
- Modern tests of Relativity Theory
  - accelerators and particles
  - gravitational lensing
  - gravitational waves (?)
  - space travel
  - GPS

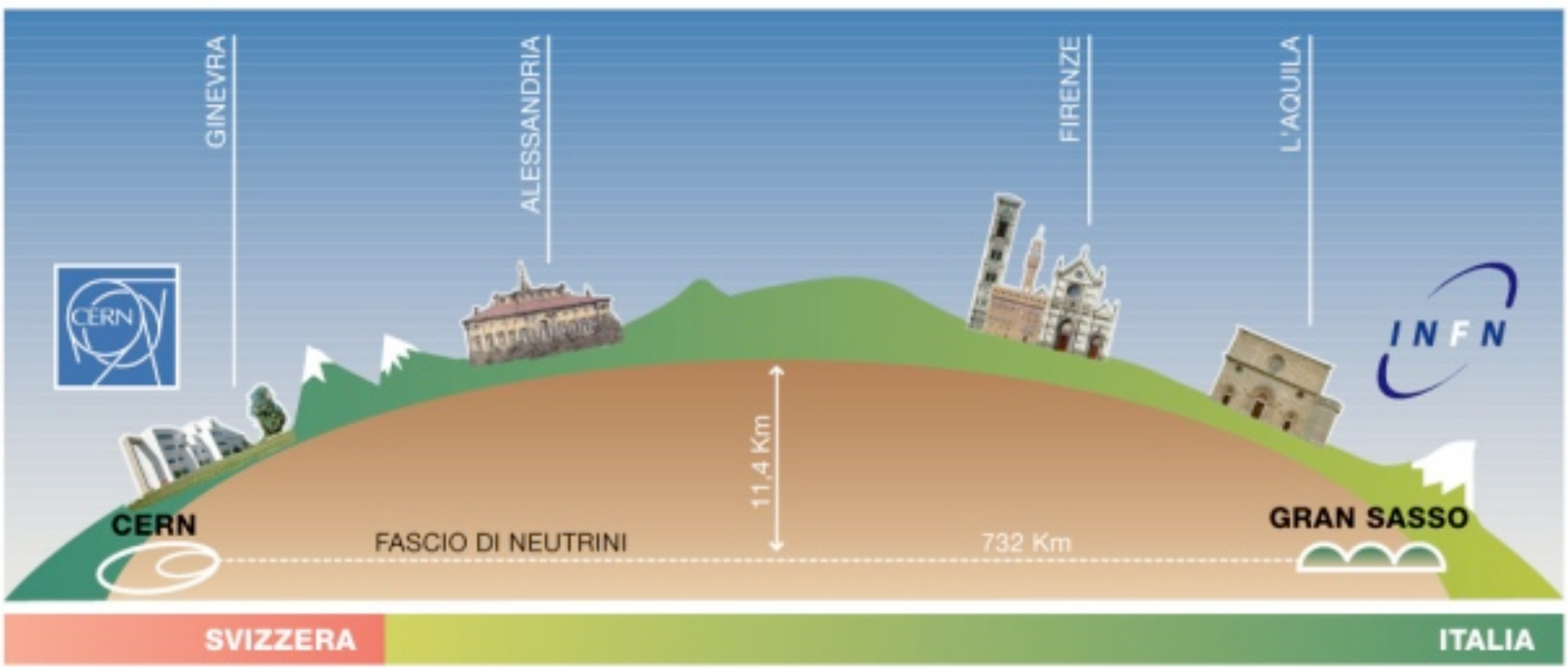






Superluminal neutrinos ?

- The Opera experiment was studying neutrino oscillations
- In September 2011, they have reported neutrinos travelling a tiny fraction faster than the speed of light
- Further investigations suggest that it might have been a timing error



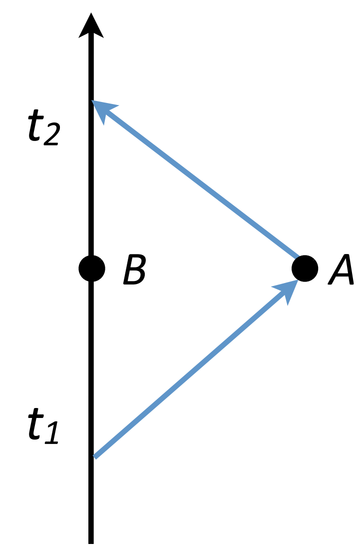
Operational definitions of Distance and Time

We take 'clocks' and 'light signals' as fundamental. A clock can be based on the vibrations of a single atom and a light signal can, in principle, be a single photon.

A non-accelerating observer moving along a straight line can use his clock and light signals to assign coordinates t and x to distant events on the line.

The observer sends out a light signal at time t_1. This is measured at an event *A* on the line and immediately transmitted back to the observer, arriving at time t_2. The times are measured on the observer's clock.

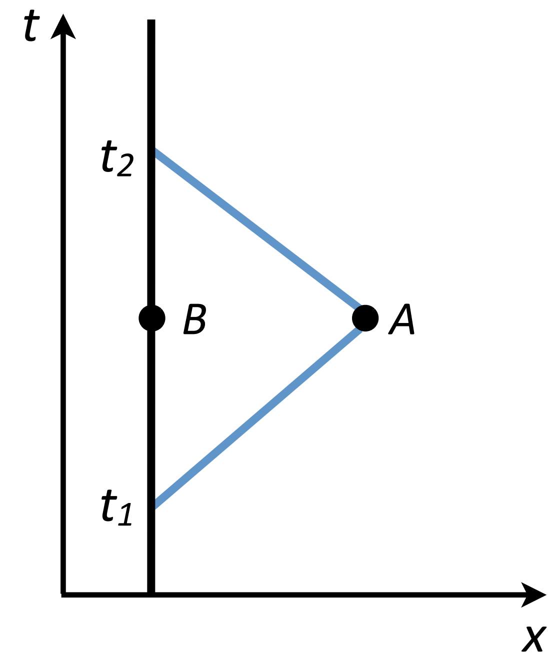
Which event *B* at the observer is simultaneous with *A* ?



Operational definitions of Distance and Time

If the velocity of photons is constant, then the outgoing and the returning journeys of the photons will take equally long and the event *B* at the location of the observer that is simultaneous to *A* is the event at time

$$t_A = t_B = \frac{1}{2}(t_1 + t_2)$$



This is the radar definition of simultaneity.

The black line in the figure is the *world line* of the observer, the blue lines are the world lines of the outgoing and returning photons.

This figure is called a space-time diagram.

It has only one space dimension and one time dimension.

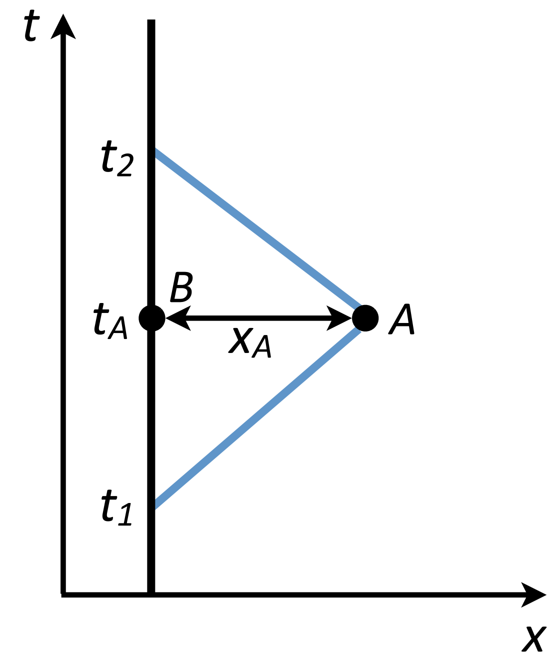
Operational definitions of Distance and Time

The distance that is assigned to the separation of B from A therefore is:

$$x_A = \frac{1}{2} c(t_2 - t_1)$$

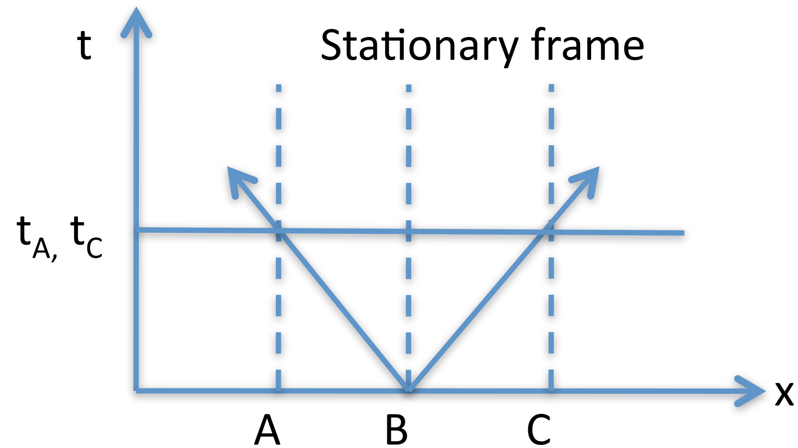
By defining distance and simultaneity in this way, any non-accelerating observer can set up a coordinate system to label any event by its radar distance x_A to his own location and the time t_A at which it happens according to the radar definition.

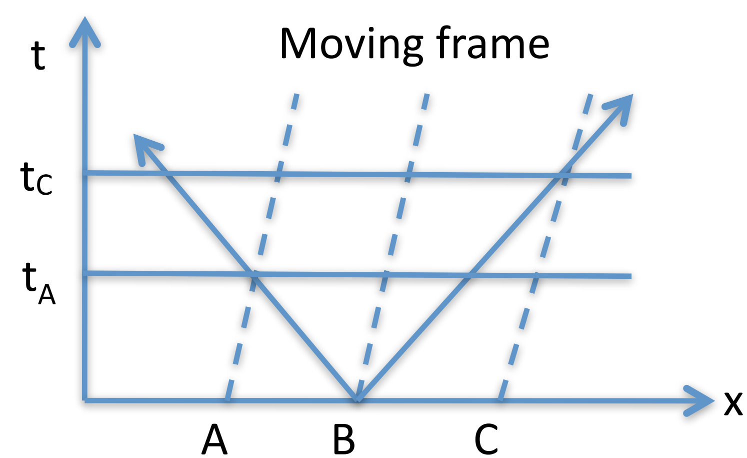
Hence we re-defined inertial coordinates; the coordinate system is an inertial system. We will always assume that the observer sets $x=0$ at his own location.



Simultaneity

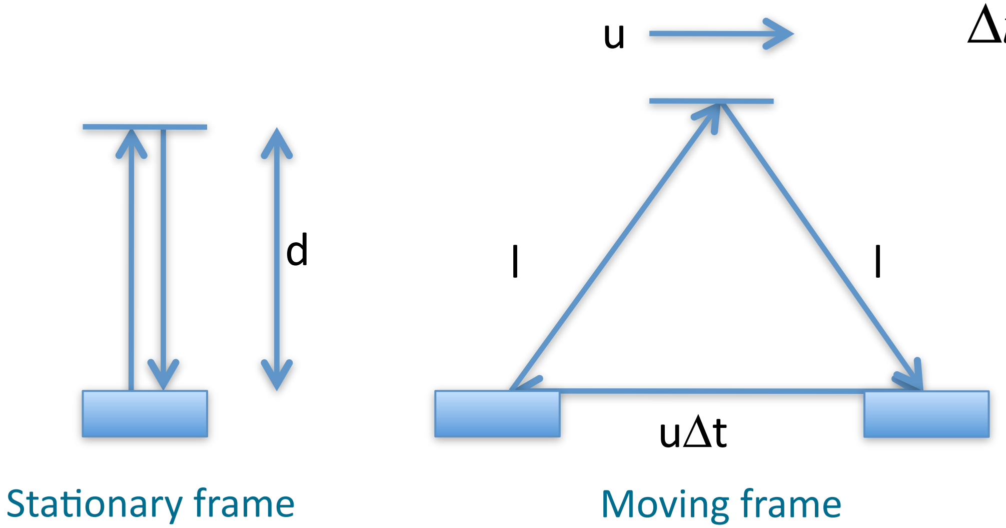
- A space-time diagram shows the motion of a particle in time and space
  - $x = x_B \pm ct$ for light emitted at B
  - In a stationary frame, A, B, C are all at fixed x positions
  - In the moving frame the positions x of A, B, C change, $x=ut$
- Two events are considered *simultaneous* if they occur at the same time
- In relativity the simultaneity of two events depends on the reference frame
  - Events are simultaneous in the stationary frame but are not simultaneous in the moving frame
  - The path of the light is the same in both frames as the speed of light is c in both frames
  - A appears to go towards the light and C appears to move away from the light.





Transformation of time intervals in Special Relativity

- We can set up an experiment where light is emitted from a source, bounces off a mirror and returns to the source
- We measure the time interval in the rest frame and in in the moving frame



In the stationary frame

$$\Delta t_0 = \frac{2d}{c}$$

In the moving frame

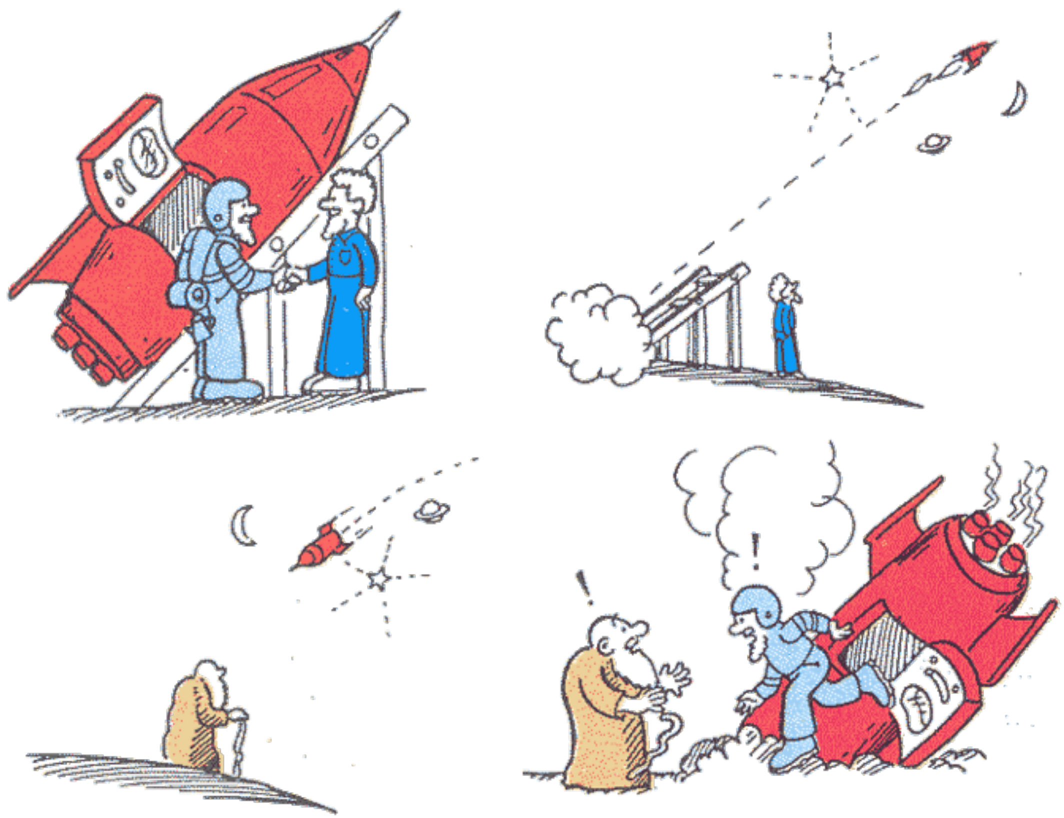
$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \gamma\,\Delta t_0$$

This effect is called time dilation.

Twin paradox

Alex and Box are twin brothers. Bob makes a journey to a distant star at a speed close to the speed of light. When he returns finds Alex much older than him.



Twin paradox

- Because Bob moves at high velocity with respect to Alex, time flows much slower for Bob

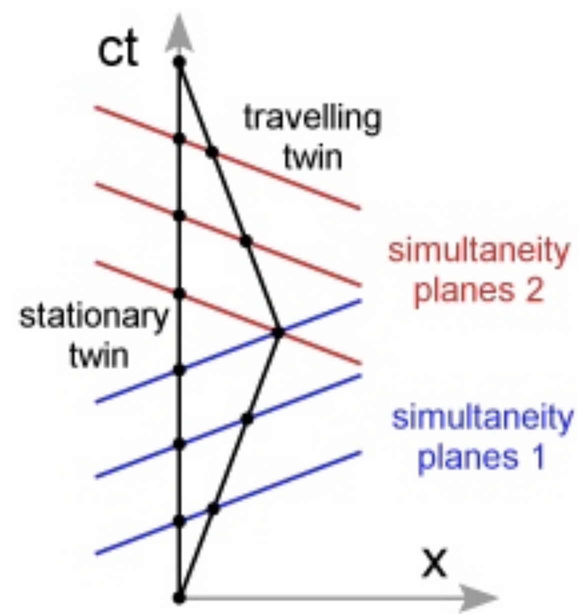
$$\Delta t_A = \gamma \Delta t_B$$

- All reference frames are equivalent, so Alex moves with respect to Bob at the same spee hence

$$\Delta t_B = \gamma \Delta t_A$$

- But, evidently, not both equations can be true, or else $\gamma=1$.

- What is actually going on ?



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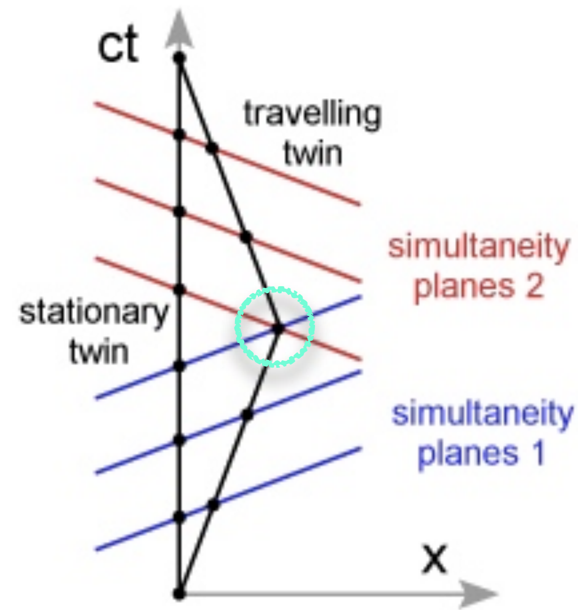


Transformation of space intervals in Special Relativity

- Similarly with time dilation, the length of a ruler measured in a frame in which it is moving is less than the length measured in its rest frame

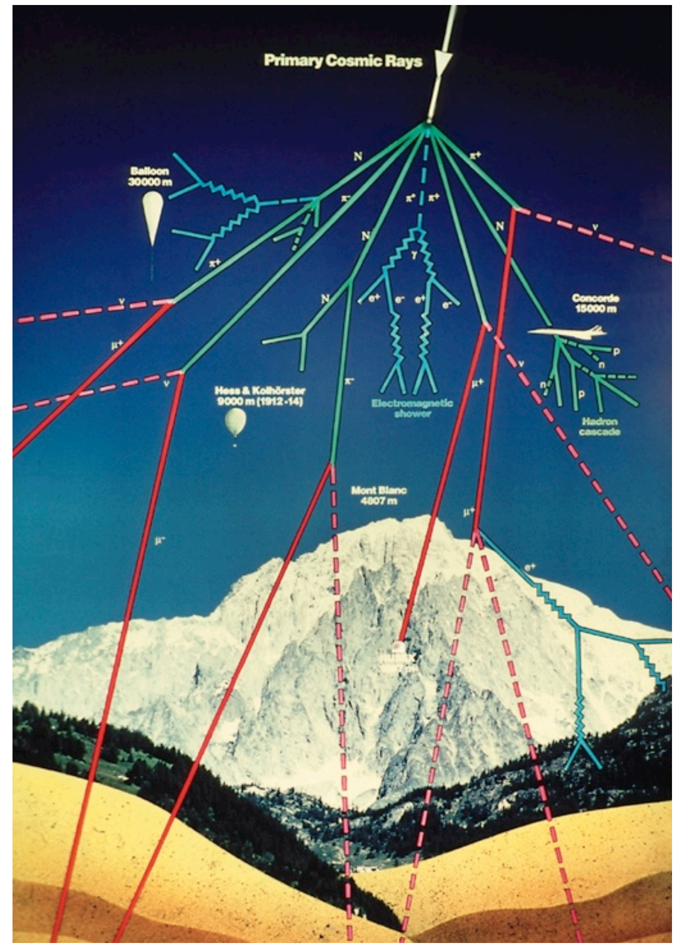
$$\Delta l = \Delta l_0 \sqrt{1-\beta^2}$$

$$\Delta l = \frac{\Delta l_0}{\gamma} \qquad \text{where} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

This effect is called **length contraction**, also known as FitzGerald-Lorenz contraction.

Muon lifetime

- Cosmic ray muons are created when highly energetic radiation from deep space interacts with atoms in the Earth's atmosphere. The initial collisions create pions which decay into muons.
- The muons are produced in the upper atmosphere, 15-20 km above sea level.
- The muon has a measured mean lifetime $\tau \simeq 2.2\ \mu s$
- If muons travels at a velocity $v=0.998c$, where c is the speed of light, will they reach ground level ?



Muon lifetime puzzle

A muon moving at 0.998 the speed of light would travel the 15km distance in a time

$$t = H/c = 15000m/(0.998\times 3\times 10^{8} m/s) \approx 50\mu s$$

so the fraction of muons reaching ground level is

$$\frac{N}{N_0} = \exp\left(-\frac{t}{\tau}\right) = \exp\left(-\frac{50}{2.2}\right) \simeq 1.3\times 10^{-10}$$

i.e. practically none will reach ground level.

However, for an observer on Earth the lifetime of the muons is

$$\tau_E = \tau\gamma = \tau/\sqrt{1-0.998^2} \approx 35\mu s$$

and then the fraction of muons reaching the Earth should be calculated with

$$\frac{N'}{N_0} = \exp\left(-\frac{t}{\tau_E}\right) = \exp\left(-\frac{50}{35}\right) \simeq 0.24 \approx 24\%$$

Four-vectors

- In three coordinates x,y,z we can define a position in space $\boldsymbol{x}=(x,y,z)$
- The distance d between two points can be defined as

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

- This distance is invariant under galilean transformations

- In relativity we need to deal with space *and* time
- We can define the Minkowski space as a 4-dimensional vector space with 3 spacial and 1 temporal dimensions $x=(ct,x,y,z)$
- The Minkowski space replaces Euclidean space as the natural framework of reality once relativity comes into play
- The corresponding distance in Minkowski space is

$$s^2 = c^2(t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$$

- This is invariant under Lorentz transformations, i.e. *Lorentz invariant*