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Study of Bhabha Scattering Contributions for the PIENU experiment

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The PiENu experiment is a precision measurement of the rate of pion decay R^{π} including its radiative components. In order to compare the result with the theoretical prediction at the 0.1% level, accurate corrections that take into account various secondary effects are necessary. We investigate the magnitude of the Bhabha Scattering contributions and their impact on the calculation of the tail fraction.

Contents

I. INTRODUCTION

The PiENu experiment aims to measure the pion branching ratio

$$R^{\pi} = \frac{\Gamma(\pi^+ \to e^+\nu_e + \pi^+ \to e^+\nu_e\gamma)}{\Gamma(\pi^+ \to \mu^{\nu}_{\mu} + \pi^+ \to \mu^+\nu_{\mu}\gamma)} \quad (1)$$

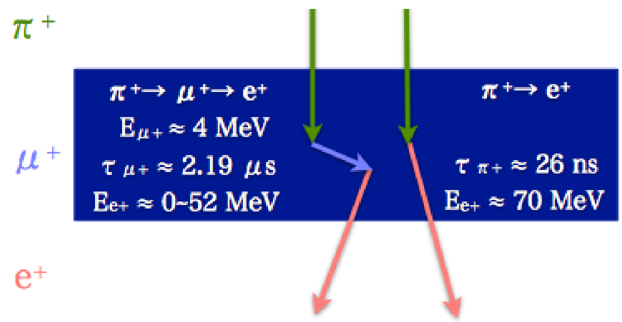

FIG. 1: Schematic of the $\pi^+ \to e^+\nu_e$ and $\pi^+ \to \mu^+ \to e^+$ processes (figure taken from ref. [1]).

with the precision necessary to compare R^{π} with the theoretical prediction at the 0.1% level.

At such precision, accurate corrections that take into account various secondary physics effects are necessary. The current experiment aims to obtain, or at least confirm, all these corrections from data in order to reduce any dependance on Monte-Carlo. In this context, Bhabha scattering of the positron might become important.

This study focuses on estimating the magnitude of these effects and their impact on the calculation of the tail fraction. Bhabha contributions to the tail correction cannot be extracted directly from the data and have to be calculated using Monte Carlo events, then the MC has to be validated by comparison with actual data.

II. TAIL FRACTION

In order to correct for the tail of the $\pi^+ \to e^+\nu_e$ (*pienu*) distribution that falls under the $\pi^+ \to \mu^+ \to e^+$ (*pimue*, or *pimunu*) distribution, the *pimue* events are suppressed with an early time cut ($7 < t < 33$ ns) and then with a total energy (E_{tot}) cut which takes advantage of the fact that the muon from the *pimunu* decay leaves an additional 4.1 MeV in the target (see FIG.1). The total energy cut is shown in FIG.2 taken from reference [1].

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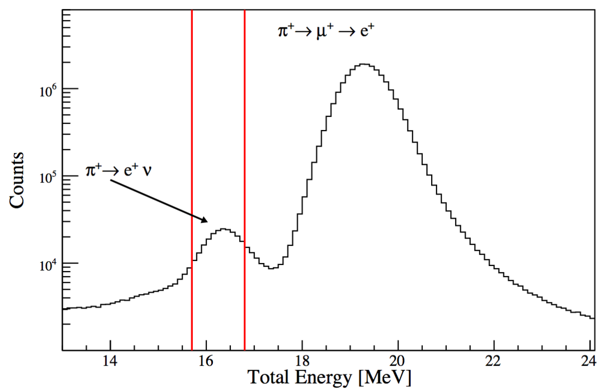


FIG. 2: E_{tot} spectrum, as shown in Fig. 5.1 from reference [1]. The vertical red lines correspond to the cut $15.7 < E_{tot} < 16.8$ MeV. See FIG.11 from Appendix 3 for comparison.

The quantity E_{tot}, on which the above cut is based, is the sum of the energies deposited in the target (Tg) and upstream counters (B1, B2, S1, S2):

$$E_{tot} = E_{B1} + E_{B2} + E_{S1} + E_{S2} + E_{Tg} \quad (2)$$

Positrons that Bhabha scatter in the target will produce an electron that deposits additional energy in the target, thus raising the energy in the E_{tot} spectrum, with the effect that such events are excluded from the selected region.

In this study, we consider only positrons that undergo Bhabha scattering *within the target volume*. Bhabha scattering downstream of the target is irrelevant in that it can not affect E_{tot} defined by Eq.(2). Bhabha scattering upstream of the target would occur for decay-in-flight (PDIF) events but this is (a) known to be a very small contribution and (b) at least partially addressed by other cuts (e.g. kink cut [1]).

III. CUTS AND EVENT SELECTION RULES

The following cuts and selection rules are used in [1] and in this analysis:

C1. **One π^+ in B1 and B2**: $(N_{B1} = 1$ & $N_{B2} = 1$ & $PID_{B1} = 211$ & $PID_{B2} = 211)$

C2. **The π^+ decays at rest, in target**: (a) $\mathbf{p}^{\pi+}_{decay} = 0$ & (b) $|z^{\pi+}_{decay}| < 4$ mm

C3. **Trigger thresholds**: $E_{T2} > 0.1$ MeV & $E_{T1} > 0.1$ MeV

C4. **WC3 radial cut**: $R_{WC3} < 60$ mm

where N_{Bx} is the number of hits in detector Bx, and PID_{Bx} is the particle id of the hit in detector Bx.

Events containing a Bhabha scattered electron can be selected at *MC truth* level with the conditions:

C5. **Bhabha scattering flag**: $E_{Bh} > 0$ & $E_{ele} > 0$

The quantity E_{Bh} is used in `SteppingAction` to flag events including `G4eIonisation` processes (i.e. ionization and energy loss by electrons and positrons) occurring in the "Target" volume (see Appendix 1). In such case, E_{Bh} will contain the energy of the positron. Condition $E_{ele} > 0$ ensures that the outgoing electron has not stopped immediately after undergoing Bhabha scattering [6].

A. Production fCut

During our investigation of the electron energy distribution we have noticed that an artificial threshold of 2 MeV was present. Looking at the definitions of the materials in the MC, we have discovered that this threshold was affecting all the materials coded in the `Materials.cc` file, but none from the G4 materials database. The problem was traced to a forgotten production cut

```
Cuts->SetProductionCut(10*mm);
```
(3)

in file `WorldConstructor.cc` [4]. This *fCut* was overriding all values set in the `PhysicsList.cc`,

```
defaultCutValue = 1.0*mm;
   fCutForGamma = 1.0*mm;
fCutForElectron = 0.1*mm;
fCutForPositron = 0.1*mm;
```
(4)

and had to be removed in order to obtain a correct MC simulation of the PiENu physics. See Appendix 2 for more details.

One should note that the Monte Carlo results shown in reference [1] were based on the incorrect cut (3), while all results presented in this report employ the correct settings (4). Results obtained with the two settings (3 and 4) are compared in Appendix 4.

IV. ANGULAR DISTRIBUTIONS

To be able to select a predominantly Bhabha subsample for a comparison between MC and data, it is important to understand the geometry of Bhabha events.

A. Positron angles

When selecting MC events where the e^+ Bhabha scatters in the target, we obtain an angular distribution peaked around 90°. This is explained by the fact that the probability of Bhabha scattering depends on the amount of target material traversed by the positron and this, due to the geometry of the target, is dependent on the angle

(θ_{e+}) of the positron with respect to the beam direction. FIG.3 shows the θ_{e+} distribution obtained from Monte Carlo.

B. Electron energy and angles

Electrons resulting from Bhabha scattering in the target are selected as shown in Appendix 1. Their angular distribution and energy spectrum are shown in FIG.4.

The scattered electrons carry little momentum and the direction of the positron is not significantly altered by the Bhabha scattering. FIG.5 illustrates this by showing the correlation between the angles of the original and Bhabha-scattered positrons.

V. TRIGGER ACCEPTANCE

Since the majority of Bhabha scattered positrons are travelling perpendicular to the beam direction, they are much less likely to give a trigger than normal events. Applying the trigger conditions C3+C4 in the Monte Carlo, we find the distribution shown in FIG.6. This requirement reduces the number of Bhabha events by a factor of ≈1/9.

FIG.6 (top) can be understood, qualitatively, as follows: the large peak at smaller angles corresponds to the case where the trigger was made by the positron. The residue above 60° is due to triggers made by Bhabha scattered electrons.

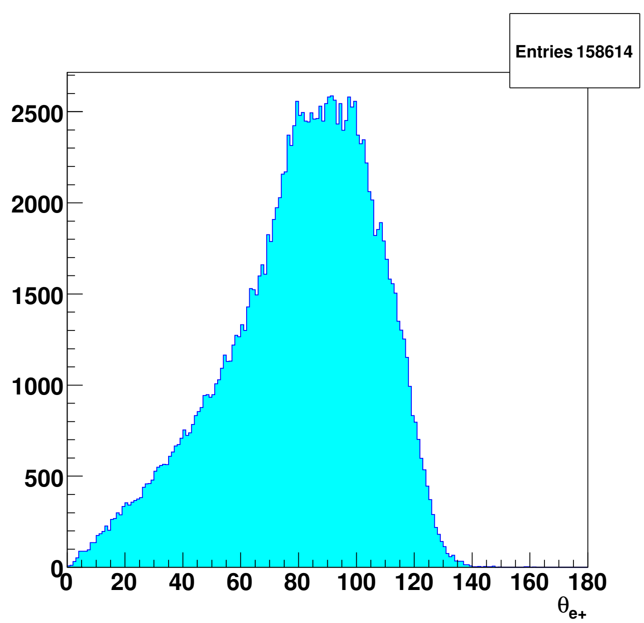


FIG. 3: Angular distribution of Bhabha scattered positrons from MC. Conditions C1+C2+C5 are applied here and θ angles are measured w.r.t. the beam direction ($\pi^+ \to e^+\nu_e$ Monte Carlo).

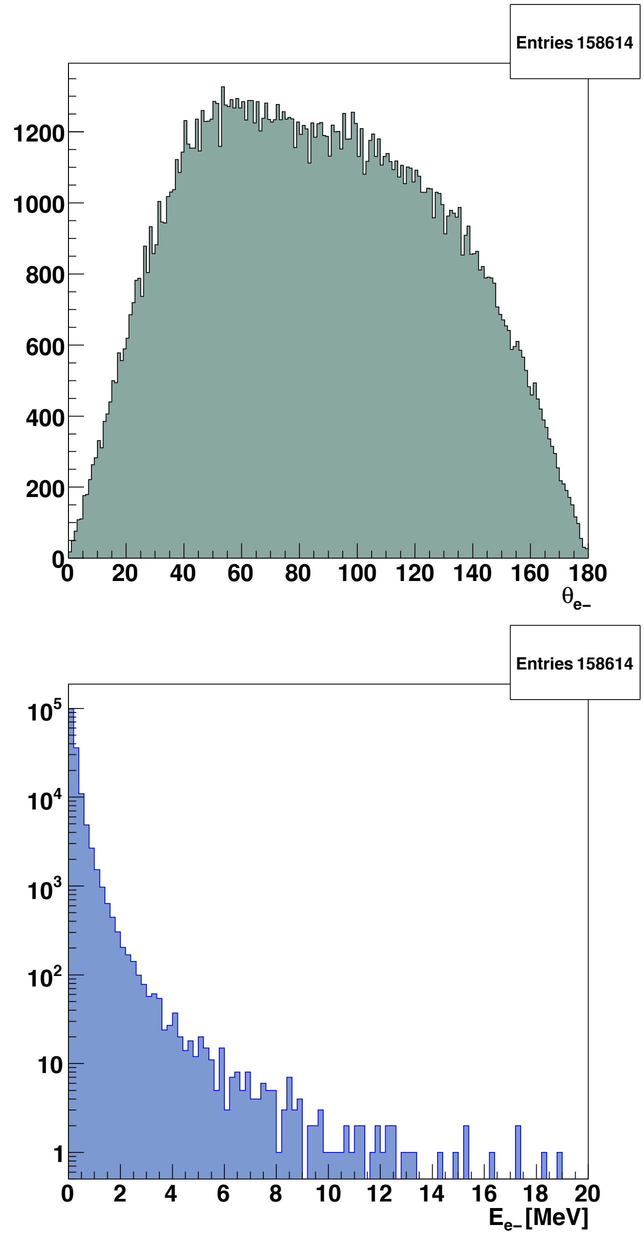


FIG. 4: Theta angle θ_{e-} and kinetic energy E_{e-} of the Bhabha-scattered electron when conditions C1+C2+C5 are applied ($\pi^+ \to e^+\nu_e$ Monte Carlo).

This interpretation can be supported by looking at the energy of the Bhabha particles in the NaI detector, shown in FIG.7. We have seen in FIG.4 that the Bhabha electron has predominantly low energies and thus we would expect that events with $\theta_{e+} > 60°$ should correspond to low energy events in the NaI (which detects the triggering electron) and events in the lower peak ($\theta_{e+} < 60°$) should correspond to higher energy events in the NaI detector (which, in this case, has detected the triggering positron that still has most of the 70 MeV).

From the $\pi^+ \to e^+\nu_e$ distributions in FIG.7 (top) one can extract a rough estimate of the percentage of

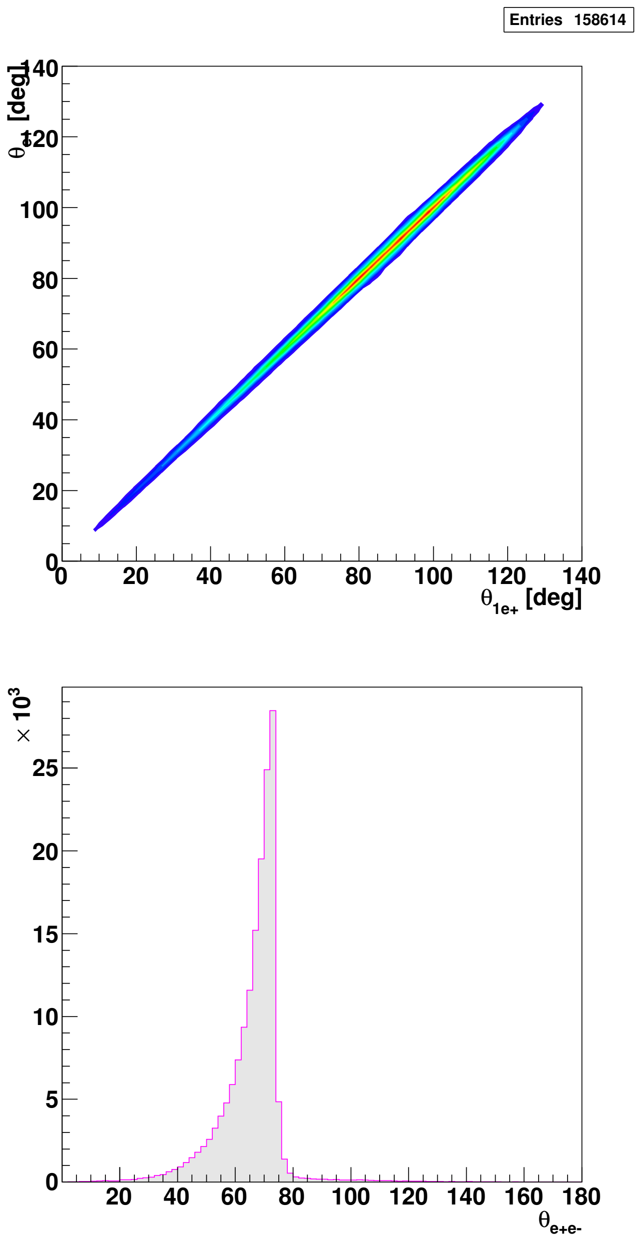


FIG. 5: **Top**: θ_{e+} angle of the scattered positron plotted versus the θ^0_{e+} angle of the positron originating from the π^+ decay in the target. Conditions C1+C2+C5 are applied here and θ angles are measured w.r.t. beam direction. **Bottom**: Angle between the Bhabha scattered $e^+$and e^- (from $\pi^+ \to e^+\nu_e$ MC truth level). Same cuts.

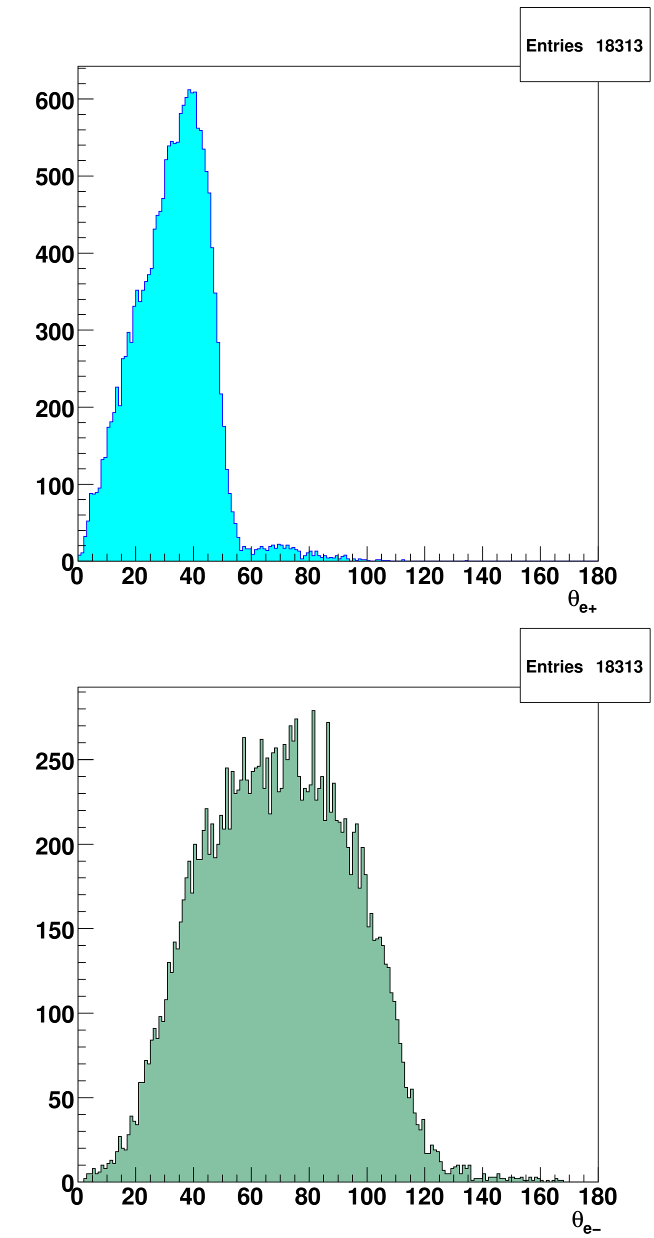


FIG. 6: Theta distribution of Bhabha-scattered positrons (top) and electrons (bottom) from $\pi^+ \to e^+\nu_e$ Monte Carlo, after acceptance and trigger conditions C1+C2+C3+C4+C5 are applied. Compare with FIG.3 and FIG.4.

Bhabha events where the trigger is given by the Bhabha electron, which was found to be $\approx 1.5\%$.

VI. BHABHA CORRECTION TO THE TAIL FRACTION

The Bhabha correction to the tail fraction is calculated using Monte Carlo events, and then the values obtained must be validated by comparison of the Bhabha effects in the MC and data.

FIG.8 shows the $(E_{NaI} + E_{CsI})$ spectrum when various cuts (explained in the caption) are applied. We define here the *tail fraction* as the ratio between the numbers of counts below and above 50 MeV. The *Bhabha correction to the tail fraction* is obtained by calculating the same ratio for Bhabha events, i.e. with condition C5 added.

A calculation done using an extended Monte Carlo

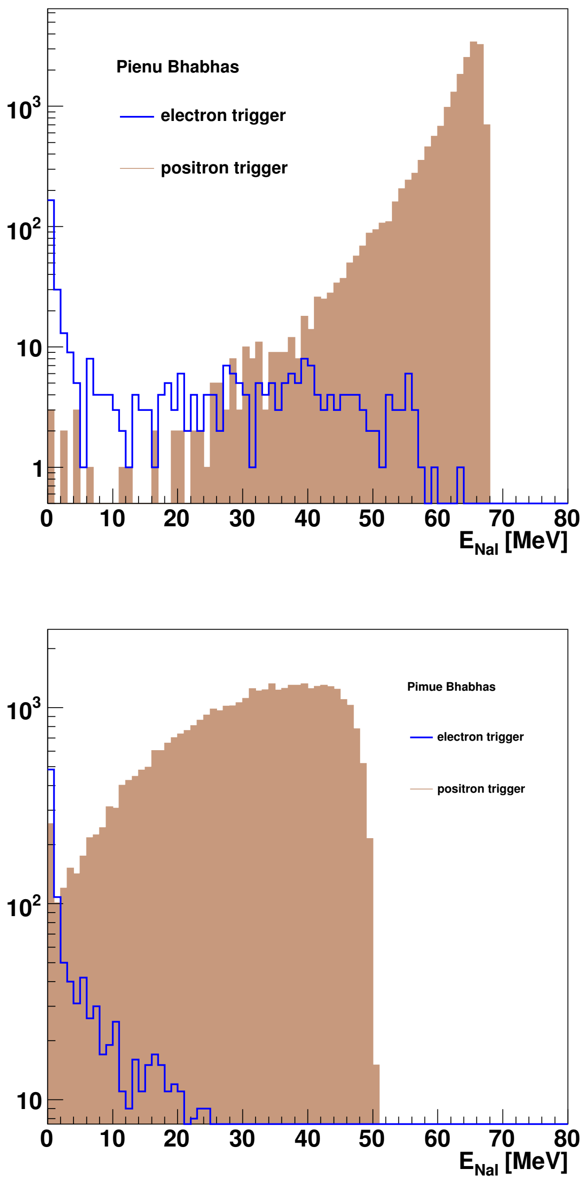


FIG. 7: Energy deposited in the NaI detector (BINA) by Bhabha-scattered particles, from $\pi^+ \rightarrow e^+\nu_e$ Monte Carlo. Here conditions C1+C2+C3+C4+C5 are combined with $\theta_{e+} < 60^\circ$ (brown), and $\theta_{e+} > 60^\circ$ (blue). See θ_{e+} distribution from FIG.6 (top). Bottom plot shows the same for $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ for comparison.

event sample [4] gives the values:

$$\begin{aligned} f_T &= 0.01904 \pm 0.00008 \\ f_T^c &= 0.00896 \pm 0.00006 \\ b_T &= 0.0101 \pm 0.0001 \end{aligned} \quad (5)$$

where by f_T we denoted the tail fraction and f_T^c is the tail fraction with E_{tot} cut, and b_T is the Bhabha correction to the tail fraction.

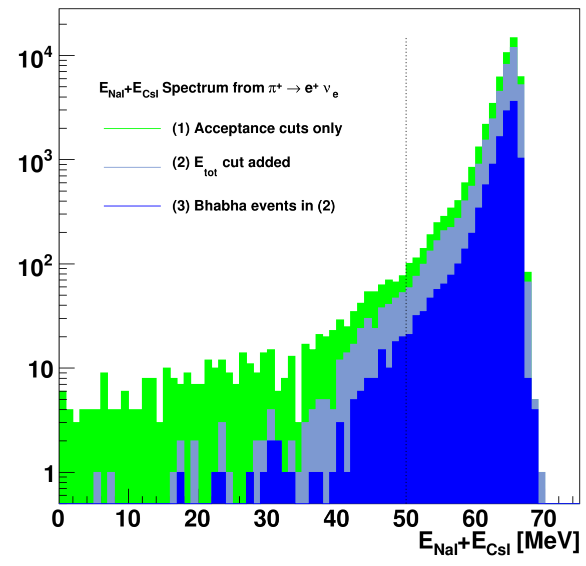


FIG. 8: Sum of the energies deposited in the NaI and CsI detectors, from $\pi^+ \rightarrow e^+\nu_e$ MC. Conditions C1+C2+C3+C4 are applied (green). For the suppressed spectrum (light blue), the condition $15.7 < E_{tot} < 16.8$ MeV is applied. The Bhabha contribution (dark blue) is obtained by adding condition C5.

For comparison, the same calculation done using the incorrect Monte Carlo production cut (3) was giving the values

$$\begin{aligned} f_T &= 0.01585 \pm 0.00007 \\ f_T^c &= 0.00887 \pm 0.00006 \\ b_T &= 0.0069 \pm 0.0001 \end{aligned} \quad (6)$$

which are significantly different.

This Bhabha correction is calculated using Monte Carlo events, so the next step is to validate our MC by making a comparison of the Bhabha effects in our simulations and the actual data.

These results should be updated and should include here details on how the the systematic uncertainty on the Bhabha correction to the tail fraction was obtained.

VII. VALIDATION OF THE MONTE CARLO

We've seen that the Bhabha correction to the tail fraction cannot be extracted directly from the data and had to be calculated using Monte Carlo events. In order to verify the obtained value, one would have to (A) find a way to select a kinematic region where the Bhabha contribution is high enough, (B) find out if this can be measured precisely enough in the data, (C) compare the MC prediction with the measurement.

Several routes were tried, by looking at:

R1. 2-dimensional plots of the deposited energies in the detectors upstream of the target versus E_{tot}

R2. Two hits in WC3 or S3 that could be a signature of Bhabha events where both the electron and positron entered the acceptance region.

R3. Hits in the upstream CsI subregions could be produced by Bhabha positrons travelling at $\theta \approx 90^{\circ}$ and entering the CsI just upstream of the veto counters.

R4. Plots of E_{Tg} vs. E_{NaI} deposited energies, that are different for Bhabha and non-Bhabha events

Several of these routes seemed promising. However, the initially observed (artificial) Bhabha separation disappeared when the correct fCut was used (see Appendix 4). Conditions R2 and R3 constrain the geometry to very particular Bhabha kinematics and hence reduce the available statistics. They did not produce significant enhancements of the Bhabha event samples, unless used in conjunction with R4 (see Appendix 5).

The best way to obtain a predominantly Bhabha events sample was found by looking at the 2-dimensional plot of E_{Tg} versus E_{NaI}, where E_{Tg} is the total energy deposited in the target volume (used in Eq.2) and E_{NaI} is the total energy deposited in the NaI detector (BINA) [4]. We have mentioned earlier why Bhabha events will have a higher energy deposit in the target, and we have seen in FIG.7 that the NaI energy deposits differ significantly between events with electron or positron triggers.

Figure FIG.9 shows scatter plots of E_{Tg} vs. E_{NaI} for Bhabha and non-Bhabha events from Monte Carlo. This analysis was done using $\pi^+ \to \mu^+ \to e^+$ events, which are more abundant in the data. Bhabha events are selected with condition C5, and to select non-Bhabhas we require that $E_{Bh} \leq 0$ in our reconstructed MC.

One can see that a cut like $E_{Tg} > 12$ MeV would select predominantly Bhabha events, and Monte Carlo generated E_{NaI} spectra for such events could be compared with the data (see FIG.10). To quantify the Bhabha enhancement obtained with such a cut, one could define the quantity

$$\eta = \left[\frac{N(C5)}{N(!C5)}\right]_{E_{Tg}>12MeV} \quad (7)$$

where $N(C5)$ is the number of Bhabha events and $N(!C5)$ is the number of non-Bhabha events enclosed in the area selected by the enhancement condition $E_{Tg} > 12$ MeV mentioned above. From the plots in FIG.9 one can calculate $\eta \approx 22$. This figure can be increased by adding constraints R2 or R3, as shown in Appendix 5. If a large enough event sample is available, these additional constraints could be combined with R4 to further enhance the Bhabhas for the purpose of comparison between MC and experiment.

Comparison with data has been carried out by plotting the NaI deposited energy for narrow slices of E_{Tg} above

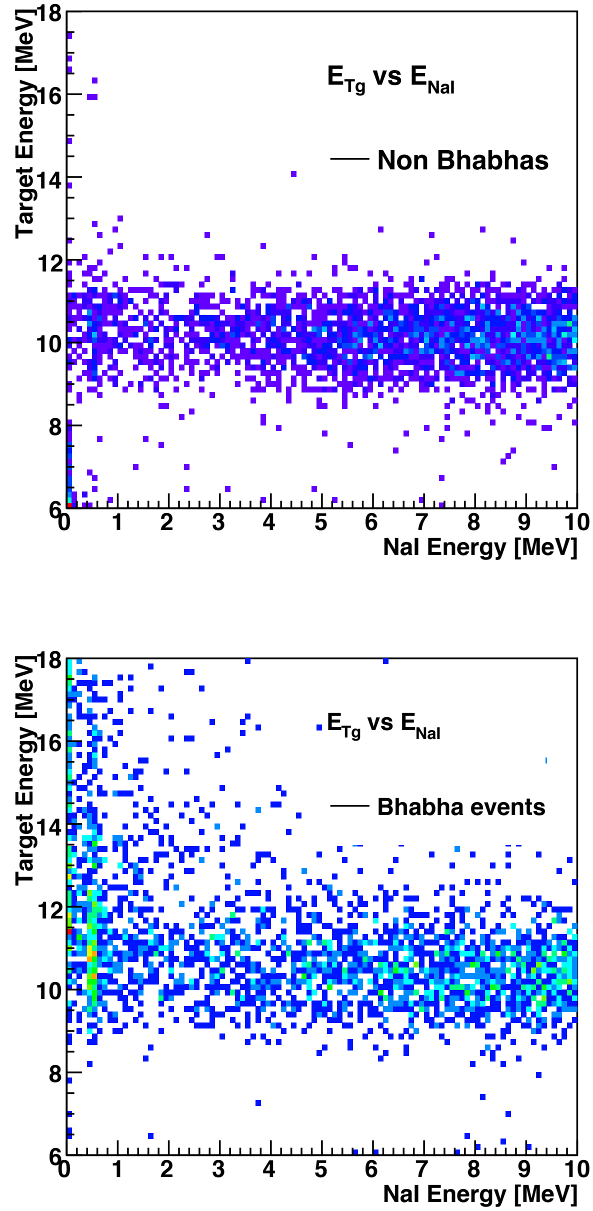


FIG. 9: Energy deposited in the target versus energy deposited in the NaI detector (BINA), for non-Bhabha events (top) and Bhabha events (bottom). Bhabha events are selected with condition C5, and non-Bhabha events with !C5. Here $\eta \approx 22$ (see Eq.7).

12 MeV [4] and Monte Carlo results show good agreement with the experiment. For completeness, we attach at the end of this report A. Sher's slides [5].

VIII. SUMMARY

The geometry and kinematics of Bhabha scattering events in $\pi^+ \to e^+\nu_e$ and $\pi^+ \to \mu^+ \to e^+$ was studied

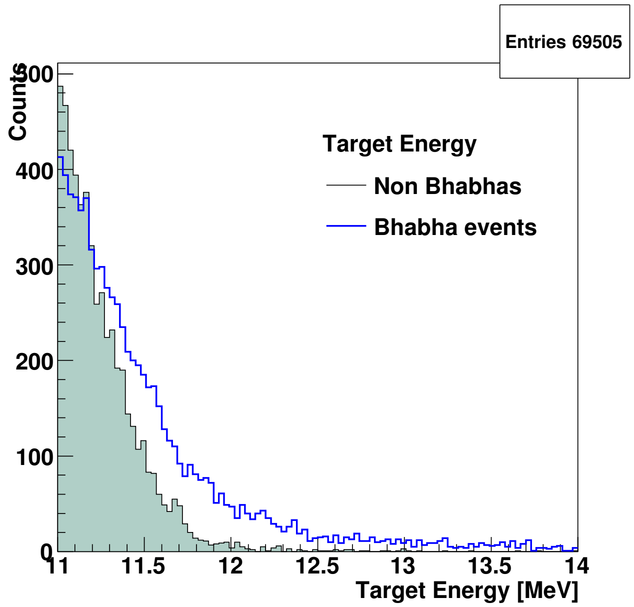


FIG. 10: Comparison of energies deposited in the target for non-Bhabha and Bhabha events. Bhabha events are selected with condition C5, and non-Bhabha events with !C5. Same $\eta \approx 22$ (see Eq.7).

using simulated events. Once Bhabha scattering was understood, its contribution to the tail fraction was estimated from Monte Carlo. What remained was to verify the predictions or our MC by comparing them with the data, within the kinematic regions of interest.

After investigating the angular distribution of the Bhabha scattered e^- and e^+, we have found that based on the geometry and energy characteristics of Bhabha scattering events it is possible to use reconstruction-level cuts to separate a predominantly Bhabha subsample in the data. This sample was then compared with our Monte Carlo and a good match was found.

Hence, our study supports applying the Bhabha correction to the tail fraction estimated from Monte Carlo to the actual data.

Appendix

1. Flagging Bhabha events in MC

Bhabha scattering events are flagged in `SteppingAction`, by assigning to E_{Bh} the energy of the positron if the process involved at a certain `step` is ”eIoni” and the volume where this occurs is ”Target”

```
if (theParticleName == "e+"
 && thePostVolume == "/pienu/Target"
 && theProcessName == "eIoni") {
    runAction->TgtBhabha(postEnergy);
 }
```

The energies and momenta of the initial positron and the outgoing positron and electron are recorded with the following conditionals:

```
if(theParticleName == "e+"
   && thePostVolume == "/pienu/Target"
   && theProcessName == "eIoni"){
  runAction->PositronFromBhabha(postEnergy,
   postMomentum);
  runAction->PositronPreBhabha(preEnergy,
   preMomentum);
}

if(theParticleName == "e-"
   && thePostVolume == "/pienu/Target"
   && theCreatorProcessName == "eIoni"
   && theTrack->GetCurrentStepNumber()==1){
  runAction->ElectronFromBhabha(prePosition,
   preEnergy, preMomentum);
}
```

The ”eIoni” physics processes are implemented in module `G4MollerBhabhaModel.cc` from the Geant4 MC simulation package [2].

2. Electron energy threshold

The energy threshold for e^- produced in the scintillator material was set at 2.19 MeV by a 1 cm ProductionCut in `WorldContructor.cc`. Here is the output from `DumpCutValuesTable()`:

```
Index : 13     used in the geometry : Yes
               recalculation needed : No
 Material : Scintillator
 Range cuts :  gamma   1 cm
               e-   1 cm
               e+   1 cm
               proton 1 cm
 Energy thresholds :  gamma   5.71952 keV
                      e-   2.18887 MeV
                      e+   2.0743 MeV
                      proton 1 MeV
```

In this case all Bhabha electrons under 2.19 MeV were absorbed in the Scintillator (i.e. never produced). When this (incorrect) setting was removed and the actual physics cuts from `PhysicsList.cc` are used, the energy threshold drops to 86.4 keV:

```
Index : 11     used in the geometry : Yes
               recalculation needed : No
 Material : Scintillator
 Range cuts :  gamma   1 mm
               e-   100 um
               e+   100 um
               proton 1 mm
 Energy thresholds :  gamma   2.40367 keV
```

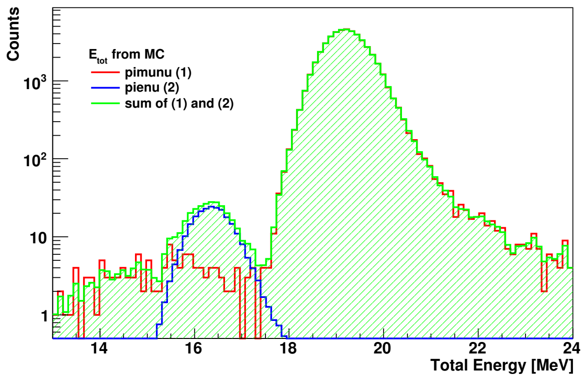


FIG. 11: Total energy plot from MC. Conditions applied: C1+C2b+C3+C4. Compare this with FIG.2.

```
e-   86.3829 keV
e+  85.2297 keV
proton 100 keV
```

Approximately 15 times more Bhabha scattered electrons are produced with this threshold, some of which can exit the target and produce a trigger.

3. Total Energy plot from MC

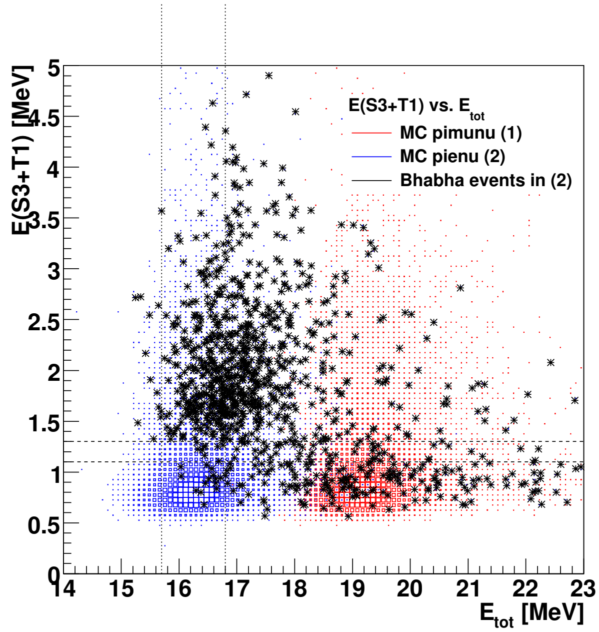
We have tried to replicate Figure 5.1 from [1] with the Monte Carlo. With no pileup and no radiative effects added, the result is shown in FIG.11. One should compare this with FIG.2.

4. Bhabha Downstream Signature

Using the 1cm MC production cut, we have noticed that Bhabha scattered electrons and positrons from events that produce a trigger had a slightly different signature in S3, T1 and T2 than 'normal' events. This is because the the extra electron will deposit an additional energy downstream of the target as well.

We have tried various combinations of S3, T1 and T2 energy deposits and we have found that the $E_{S3} + E_{T1}$ gave the best separation, shown in FIG.12 (top).

This seemed a promising avenue until we have discovered the incorrect electron energy threshold in the MC. And once the correct fCut (4) is used in the Monte Carlo, the separation completely disappears, as seen in FIG.12 (bottom).



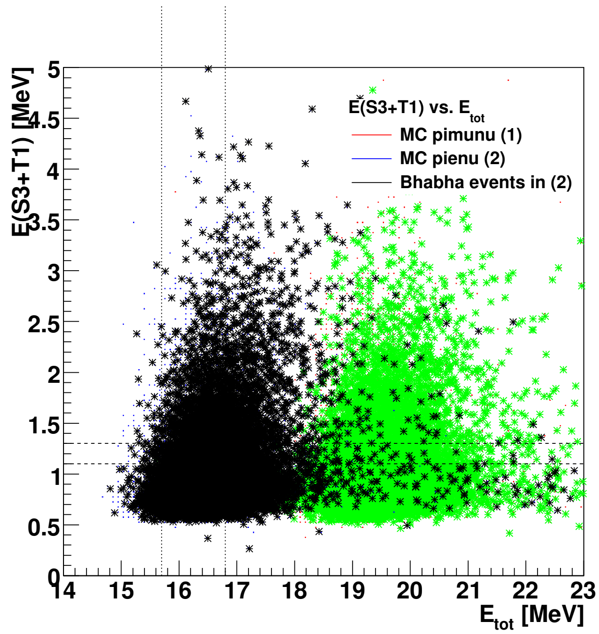


FIG. 12: **Top:** Two-dimensional plot of $E_{S3}+E_{T1}$ versus E_{tot} from MC. Conditions applied: C1+C2+C3+C4. The vertical dotted lines correspond to the cut from FIG.2. The $\pi^+ \to \mu^+ \to e^+$ events are shown in red, $\pi^+ \to e^+\nu_e$ in blue and Bhabha events from $\pi^+ \to e^+\nu_e$ (selected by adding condition C5) are drawn in black. The proportion of Bhabhas outside the E_{tot} cut is $\approx 70\%$. **Bottom**: same, when the correct fCuts (4) are applied - no separation is observed. Green markers here show Bhabha events in $\pi^+ \to \mu^+ \to e^+$.

5. Bhabha kinematic subregions

Efforts were made to obtain a Bhabha events sample as pure as possible. In theory, this could be achieved by further constraining the kinematics to preferentially select events that contain both the electron and the positron.

Two hits in S3 that could be a signature of Bhabha events where both the electron and positron entered the acceptance region (selected by the radial cut C4).

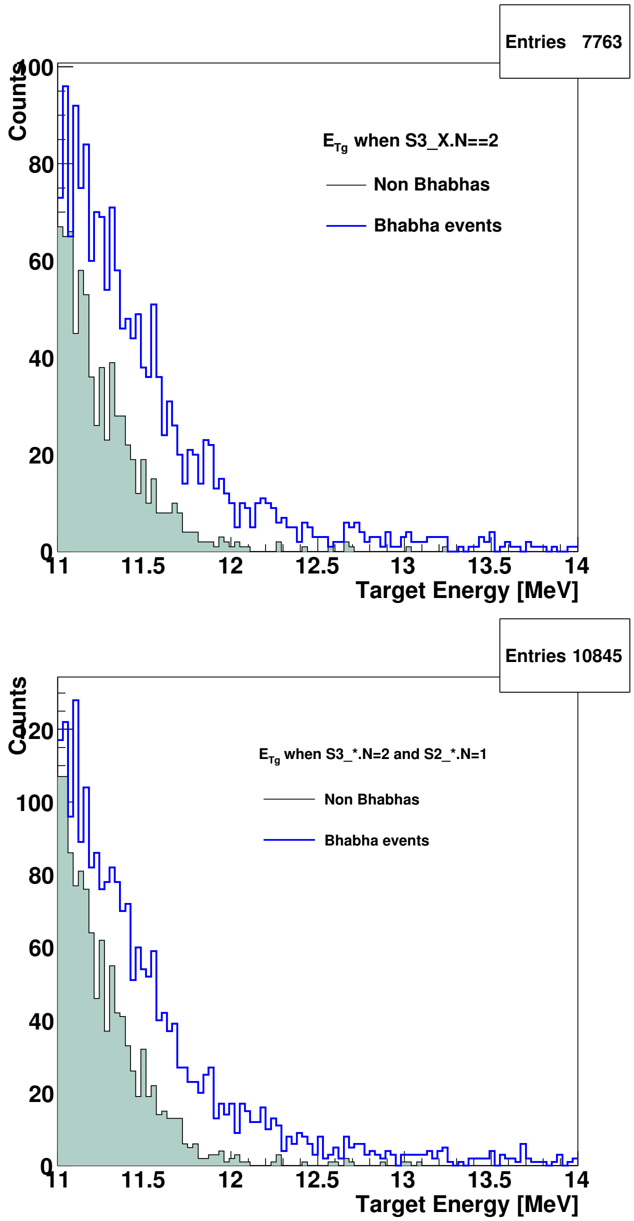


FIG. 13: E_{Tg} spectra for Bhabha and non-Bhabha subsamples from MC. Conditions applied: C1+C2b+C3+C4. **Top:** Two hits in the X plane of S3 are required: $\eta \approx 25$. **Bottom:** We require here two hits in either one of the S3 planes but only one hit in S2: $\eta \approx 29$ Compare these plots with FIG.10.

Based on the characteristic angular distributions of Bhabhas this type of requirement would select a subset of Bhabha kinematics, but if the accidentals could be dealt with, it could provide a much cleaner Bhabha sample to compare with data. FIG.13 illustrates the effects of such cut on the E_{Tg} spectrum.

Another idea was to look at the upstream CsI subregions for hits produced by Bhabha positrons travelling at $\theta_{e+} \approx 90^\circ$ and entering the CsI just upstream of the veto counters[7]. FIG.13 illustrates the effect of this requirement on the E_{Tg} spectrum.

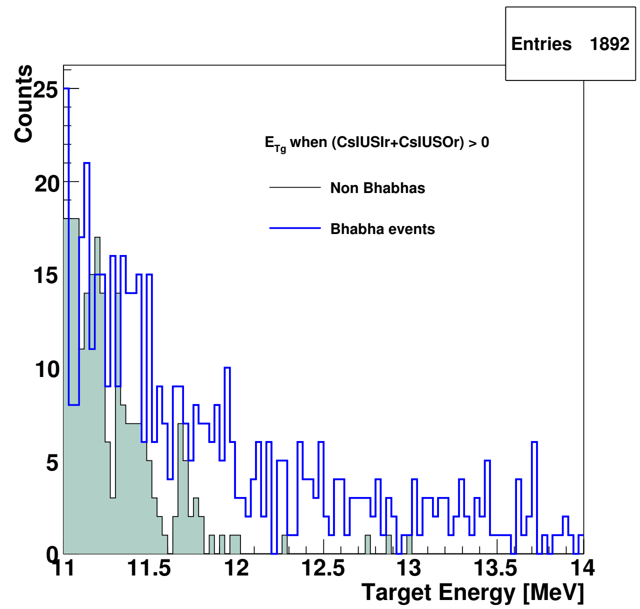


FIG. 14: E_{Tg} spectra for Bhabha and non-Bhabha subsamples from MC. Here we apply C1+C2b+C3+C4 and require that there's a hit in CsIUSI or CsIUSO: $\eta \approx 50$. Compare this with FIG.13.

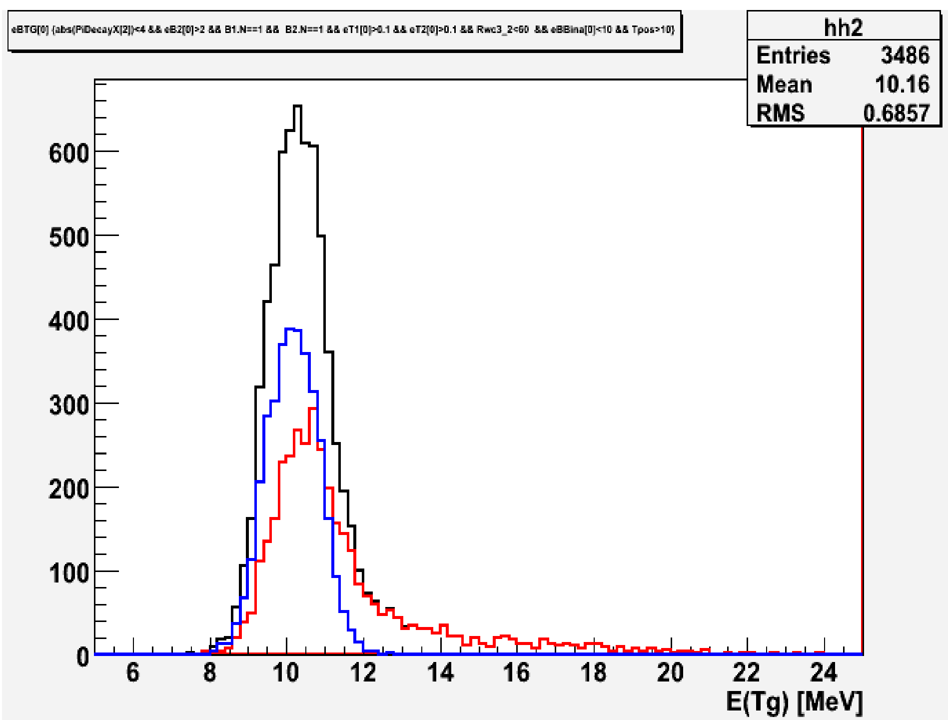
One could notice that η can be almost doubled by choosing Bhabha-specific geometries. The η values presented above should be confirmed with higher statistics. If one has a large enough event sample, these additional cuts could be used to further enhance the Bhabhas for the purpose of comparison between Monte Carlo and experiment.

[1] C. Malbrunot, Ph.D. Thesis, pienu.triumf.ca
[2] Geant4 Physics Reference Manual, geant4.cern.ch
[3] The PiENu Collaboration, MC Simulation and Reconstruction package, pienu.triumf.ca
[4] A. Sher, private communication, Mar 19, 2013
[5] A. Sher, see attached Bhabha Monte Carlo validation slides, Apr 17, 2013
[6] else E_{ele} is left to its negative initialisation value -10000 in our MC
[7] CsIUSI (upstream inner segment) and CsIUSO (upstream outer segment)

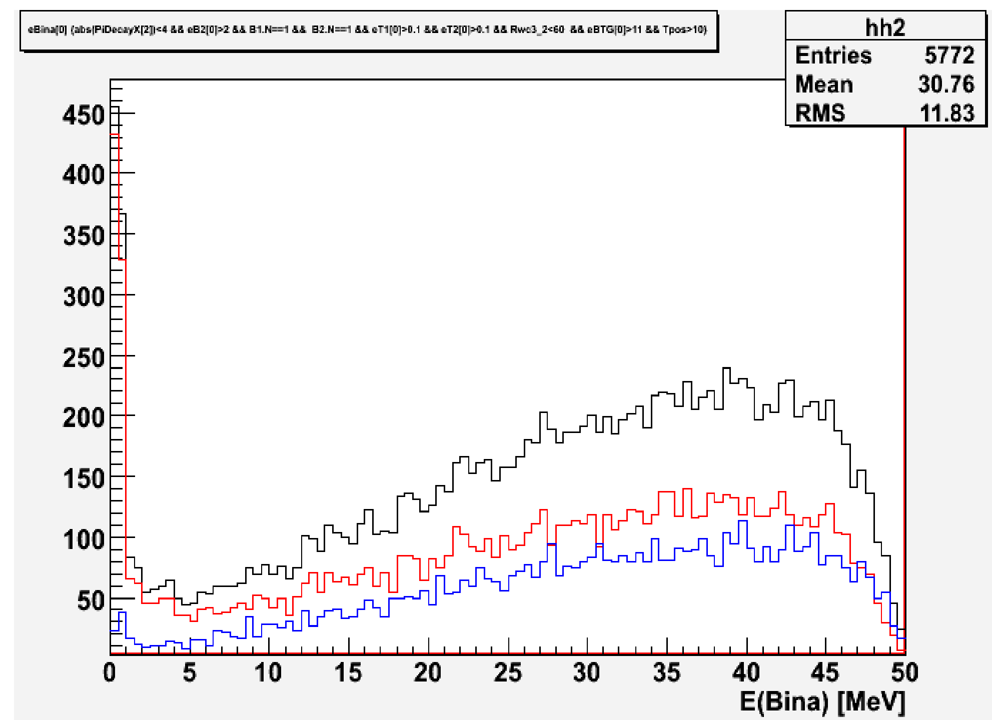
Bhabha MC validation

IDEA: Use E(Tg) and E(Bina) to **enhance** Bhabha events:

E(Bina)<10 MeV

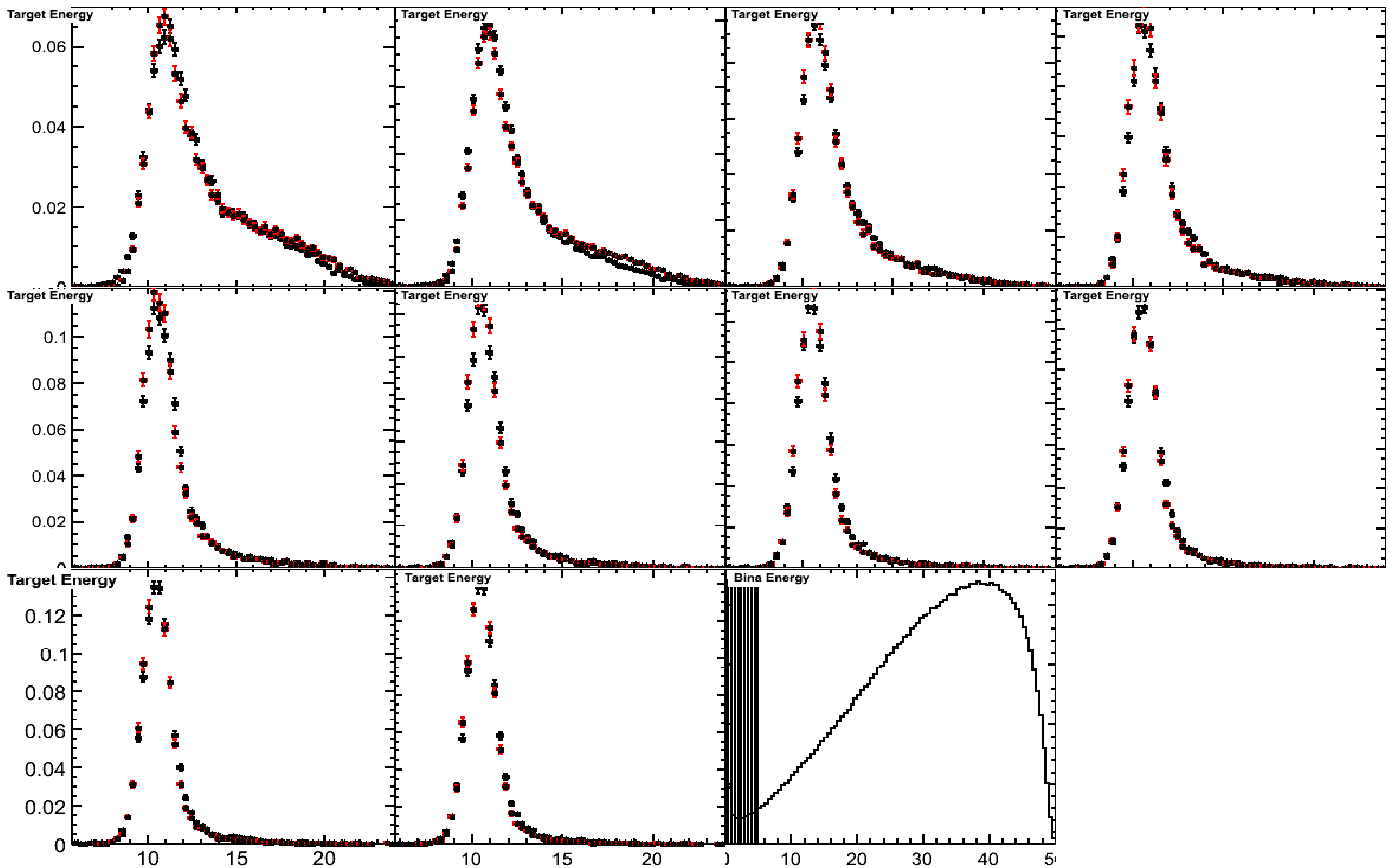


E(Tg)>11 MeV

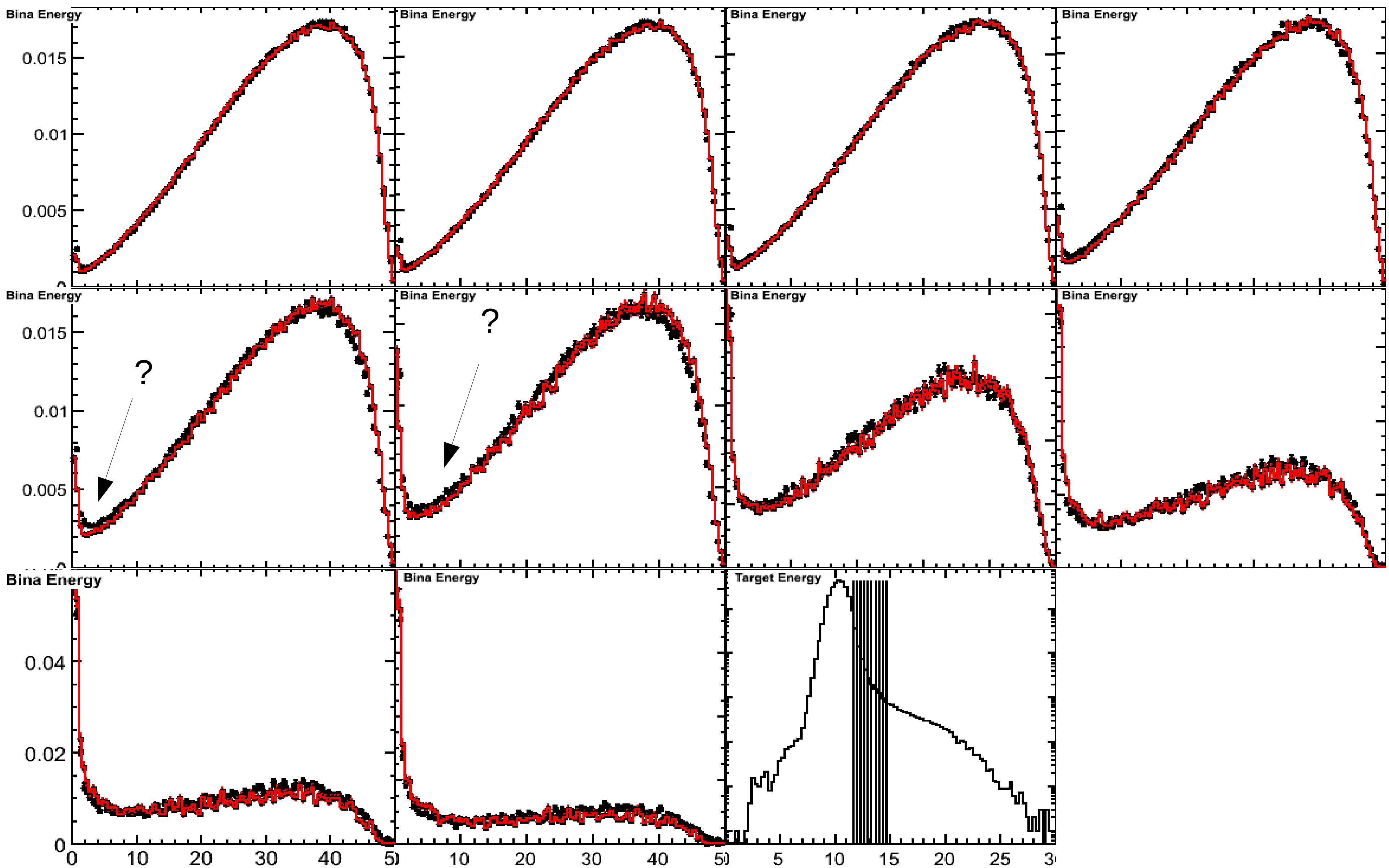


BLACK: All events RED: Bhabha events; BLUE: Regular events

DATA/MC comparisons



DATA/MC comparisons



Conclusion

- Fraction of Bhabha's can be increased using E(TG) and E(Bina)
  Low E(Bina) enhances Bhabha events, as well as high E(TG).

- Agreement between Data and MC looks good (promising), though there are some discrepancies in the Bina energy spectrum. This has to be studies further and agreement has to be quantified in terms of the Bhabha correction error.