1. General 2 state system:

   (a) Since $|s_1>$ and $|s_2>$ are the eigenstates of the operator R they must be a complete set of states. The eigenstates of the operator R can be expressed as a complete set of states:

   $$|s_a>= c_1|s_1> +c_2|s_2> .$$

   For the complete set of states, they must be normalized so that

   $$\sum_i^2 c_i^2 = 1 = c_1^2 + c_2^2.$$

   Since $\sin\theta^2 + \cos\theta^2 = 1$ the coefficients can be described in terms of one parameter, θ, called the *mixing angle.*

   (b) The properties are that the eigenstates must be othogonal and normal. Hence, $|s_b>$ must also be expressed in terms of $\sin\theta$ and $\cos\theta$ but from the orthogonality of the states, there must be sign change. Try

   $$|s_b>= -\sin\theta|s_1> + \cos\theta|s_2> .$$

   Then

   $$\begin{aligned}
   <s_a|s_b> &= (<s_1|cos\theta + <s_2|\sin\theta)(-\sin\theta|s_1> + \cos\theta|s_2>) \\
   <s_a|s_b> &= (<s_1|cos\theta + <s_2|\sin\theta)(-\sin\theta|s_1>) \\
   &+ (<s_1|cos\theta + <s_2|\sin\theta)(\cos\theta|s_2>) \\
   &= (<s_1|s_1> cos\theta(-sin\theta) + <s_2|s_1> \sin\theta(-\sin\theta) \\
   &+ (<s_1|s_2> cos\theta^2 + <s_2|s_2> \sin\theta(\cos\theta) \\
   &= 0
   \end{aligned}$$

   because $<s_1|s_2>= 0$,$<s_1|s_1>=<s_2|s_2>= 1$ by the othogonality of and normalization of the eigenstates. Check normalization:

   $$<s_a|s_a>= (cos\theta|s_1> + \sin\theta|s_2>)(cos\theta|s_1> + \sin\theta|s_2>) = 1$$

   where cross terms in sin and cos vanish by orthogonality of $|s_1>$ and $|s_2>$. Similarly $<s_b|s_b>= 1$.

The matrix equation is then:

$$\begin{pmatrix} s_a \\ s_b \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

This equation represents a two dimensional rotation of a vector between two coordinate systems with unit vectors (s_1, s_2) to (s_a, s_b). The eigenstates correspond to vectors lying along the axis in the respective system but then lies at some skew angle denoted by θ in the other system.

(c) Invert the above matrix:

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} s_a \\ s_b \end{pmatrix}$$

(d) Cabbibo matrix and angle: Here the decay eigenstates d' and s' are mixtures of the flavour eigenstates d and s. With $|d>$ defined by d and similiary for s, then using the matrix notation from above, we have

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Here the angle is the Cabbibo angle and represents the mixture of d and s mass eigenstates of the mass operator in the d' eigenstate of the weak decay. The physical quantities one measures are the masses or the decays. The value of the Cabbibo angle is determined experimentally.

(e) The mass and CP operators in the Kaon system.
Here

$$C|K^0>= C|d\overline{s}>= -|\overline{d}s>= -|\overline{K^0}>$$
$$C|\overline{K^0}>= C|\overline{d}s>= -|d\overline{s}>= -|K^0>$$

and

$$P|K^0>= P|d\overline{s}>= -|d\overline{s}>= -|K^0>$$
$$P|\overline{K^0}>= P|\overline{d}s>= -|\overline{d}s>= -|\overline{K^0}>$$

so that

$$CP|K^0>= C(-|K^0>) = |\overline{K^0}>$$

$$CP||\overline{K^0}>= C(-|\overline{K^0}>= |K^0>$$

The mass eigenstates are also a P eigenstate but not C or CP eigenstates.

We can see from the last two equations that a sum of the states gives the same functions on both sides of the equation. This is also true of the difference. But from the derivation in terms of angles we know that the coefficients have to add to one or be expressed by an angle. We thus want to have a state like $|K^0> +|\overline{K^0}>$ and another like $|K^0> -|\overline{K^0}>$. Let

$$|K_1^0>\equiv \cos\theta|K^0> + \sin\theta|\overline{K^0}>$$

so that

$$\begin{aligned} CP|K_1^0> &\equiv CP(\cos\theta|K^0> + \sin\theta|\overline{K^0}>) \\ &= \cos\theta|\overline{K^0}> + \sin\theta|K^0>) \end{aligned}$$

If $\cos\theta = \sin\theta$ we obtain

$$\begin{aligned} CP|K_1^0> &= \cos\theta(|\overline{K^0}> +|K^0>) \\ &= \frac{1}{\sqrt{2}}(|\overline{K^0}> +|K^0>) \end{aligned}$$

and $\theta = 45^o$.