1. Quantum mechanical systems with N states, where N is a small number, have many applications in particle physics. Consider $N = 2$ which describes a two state system. In this system, let the eigenstates, $|s_a>$ and $|s_b>$, of the operator, O, be labelled by their eigenvalues s_a and s_b. Let the eigenstates of another operator R be $|s_1>$ and $|s_2>$. The relation of one set of states to the other can be described by a transformation matrix, M.

   (a) Explain why it it is true that

   $$|s_a>= c_1|s_1> + c_2|s_2>,$$

   where c_1 and c_2 are constants. Describe the relationship the constants must have and show that this can be written in terms of a function of one parameter, θ so that

   $$|s_a>= cos\theta|s_1> + \sin\theta|s_2> .$$

   What is θ called?

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   (b) Using the properties of the eigenstates, determine an expression for $|s_b>$. Write the relationship of the state labelled by s_a and s_b as a matrix equation with the matrix acting on s_1 and s_2 and comment on the physical interpretation.

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   (c) Determine the expressions for $|s_1>$ and $|s_2>$ in terms of $|s_a>$ and $|s_b>$.

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   (d) Discuss how this result applies to the down and strange quark mass eigenstates and the weak decay eigenstates, relating the matrix equation derived above to the physical quantities and commenting on their values and meanings.

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(e) The C operator changes a quark into an antiquark and the P operator changes the parity of the system. Given that the parity of a K^0 and of a $\overline{K^0}$ is negative, show the action of these operators and of the product of the operators, CP, on each of these particles. Indicate if the mass eigenstates of the neutral kaon are C and P eigenstates. Form an eigenstate for the CP operator from the neutral kaon system and determine the value of θ.

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